2018

MATHEMATICS

(Major)

Paper : 6.5

(Graph and Combinatories)

Full Marks : 60

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. Answer the following questions : 1×7=7

(a) Write the multiplicative rule principle of combinatories.

(b) In how many ways can five examinations be scheduled in a week so that no two examinations are scheduled on the same day, considering Sunday as a holiday?

(c) In how many ways can a committee of 5 persons be formed from 6 men and 4 women so as to include 3 men and 2 women?

(d) Draw a simple graph having four vertices each of degree 2.

(e) Draw complete graph K_4.

(f) What is the length of a path?

(g) How many edges of a tree are having n vertices?

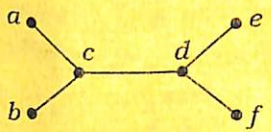
2. Answer the following questions : 2×4=8

(a) A bag contains six white marbles and five red marbles. Find the number of ways that four marbles can be drawn from the bag if they must be the same colour.

(b) How many vertices are there in a graph with 15 edges, if each vertex is of degree 3?

(c) Show that there is only one path between every pair of vertices in a tree.

(d) Find the radius and diameter of the tree shown below and show that diameter in a tree is not necessarily double of its radius :



3. Answer the following questions :

(a) Give combination proof of the following identities : 2+3=5

(i) $C(n, r) = C(n, n-r)$

(ii) $C(n+1, r) = C(n, r) + C(n, r-1)$

(b) There exists no simple graph corresponding the following degree sequences :

0, 2, 2, 3, 4

Justify the above statement. 5

(Continued)

Or

Prove that for any graph G with six vertices, G or $\overline{G}$ contains a triangle.

(c) Let v be a point of a connected graph G. Then prove that the following statements are equivalent : 5

(i) v is a cut point of G.

(ii) There exist points u and w distinct from v such that v is in every $u-w$ path.

(iii) There exists a partition of the set of points $V-\{v\}$ into subsets U and W such that for any point $u \in U$ and $w \in W$, the point v is on every $u-w$ path.

Or

Let G be a connected graph with at least three points. If G is a block, then prove that every two points of G lie on a common cycle.

4. Answer any *one* part :

(a) For any graph G, prove that

$$K(G) \leq \lambda(G) \leq \delta(G)$$

The symbols have their usual meaning. Also show that the maximum vertex connectivity of a graph G with n vertices and e edges $(e > n-1)$ is the integral part of the number, $\frac{2e}{n}$. 7+3=10

(Turn Over)

(b) State and prove Menger's theorem on graph. 10

5. Answer any *one* part :

(a) Prove that a connected graph is Eulerian if and only if every vertex of G has even degree. 10

(b) (i) If G is a simple graph with n vertices $(n \geq 3)$ and if $\deg(v) + \deg(w) \geq n$ for every pair of non-adjacent vertices v and w, then prove that G is Hamiltonian. 7

(ii) Under what conditions on r and s does the complete bipartite graph $K_{r,s}$ have a Hamiltonian circuit? 3

6. Answer any *one* part :

(a) (i) Find the number of integers between 1 and 250 that are divisible by any of the integers 2, 3, 7. 5

(ii) Find the number of integral solutions of the equation $x + y + z = 18$ with the conditions that $x < 7$, $y < 8$ and $z < 9$. 5

(b) (i) Find the number of non-negative solution of $x + y + z = 18$ with the conditions that $x \geq 3$, $y \geq 2$, $z \geq 1$. 5

(ii) What is the probability that exactly one cell is empty if ten identical balls are distributed randomly into five distinct cells? 5

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