2018

STATISTICS

(Major)

Paper : 6.1

(**Statistical Inference—2**)

Full Marks : 60

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. Choose the correct answer : 1×7=7

(a) Suppose you want to test $\theta = \theta_0$ against $\theta = \theta_1$ with w as critical region and A as acceptance region, then $P_{\theta_0}(A)$ is

(i) the probability of a correct decision

(ii) the probability of an incorrect decision

(iii) the probability of type I error

(iv) None of the above

(b) As the same symbols in **1**(*a*), $P_{\theta_1}(w)$ is

(*i*) the probability of type II error

(*ii*) the probability of type I error

(*iii*) the power of the test

(*iv*) None of the above

(c) Suppose a random sample of size n is taken from a normal population with mean μ and variance σ^2 if $\bar{x}$ is the sample mean, then the 99% confidence interval for μ is

(*i*) $\bar{x} \pm 1\cdot 96 \frac{\sigma}{\sqrt{n}}$

(*ii*) $\bar{x} \pm 2\cdot 58 \frac{\sigma}{\sqrt{n}}$

(*iii*) $\bar{x} \pm 2 \frac{\sigma}{\sqrt{n}}$

(*iv*) None of the above

(d) Empirical distribution function is based on

(*i*) sample values

(*ii*) population values

(*iii*) both population and sample values

(*iv*) None of the above

(e) For the Wilcoxon signed rank test, we assume that

(*i*) the population is not symmetric

(*ii*) the population is symmetric

(*iii*) the population is sometimes symmetric and sometimes not

(*iv*) None of the above

(f) Neyman-Pearson test gives us the

(*i*) best critical region

(*ii*) worst critical region

(*iii*) sometimes best and sometimes worst critical region

(*iv*) None of the above

(g) If for a normal distribution, the hypothesis specifies the mean but not the variance, then it is a case of

(*i*) composite hypothesis

(*ii*) simple hypothesis

(*iii*) alternative hypothesis

(*iv*) None of the above

2. Answer the following questions : 2×4=8

(a) Define the most powerful test.

(b) Define the Kolmogorov-Smirnov statistic.

(c) State the Neyman-Pearson lemma.

(d) Define the confidence interval.

3. Answer any *three* of the following questions : 5×3=15

(a) Explain briefly the likelihood ratio test. How is this approach different from Neyman-Pearson lemma? 4+1=5

(b) An urn contains 10 balls of which θ are blue (and the rest of the balls being of red and white colours). Suppose we take a sample of 3 balls and reject H_0 if all the three balls drawn yield blue balls. Calculate the probabilities of type I and type II errors, assuming sampling was done without replacement.

(c) Let $X_1, X_2, \cdots, X_n$ be a random sample from $N(\theta, \sigma^2)$, where variance σ^2 is supposed to be known. Find the confidence limits to the population mean θ with confidence coefficient $(1-\alpha)$.

(d) Write a note on Kolmogorov-Smirnov one sample statistic.

(e) Write a note on run test.

4. Answer any *three* of the following questions : 10×3=30

(a) (i) Write a note on median test. 5

(ii) Define the following : 2½+2½=5

(1) Uniformly most powerful test

(2) Type I and type II errors

(b) Find the most powerful and uniformly most powerful regions in taking random sample from a normal distribution with mean unknown but variance known. 8+2=10

(c) (i) Explain why one should go for confidence interval instead of point estimation. 5

(ii) If $x \geq 1$ is the critical region for testing $\theta = 2$ against alternative $\theta = 1$, on the basis of a single observation from the population

$$f(x, \theta) = \theta\, e^{-x\theta};\ 0 \leq x < \infty$$

obtain the values of type I and type II errors. 5

(d) Write an explanatory note on Mann-Whitney test. 10

(e) (i) Compare and contrast between Kolmogorov-Smirnov one sample test and chi-square test. 5

(ii) Enumerate the steps involved in testing mean of a normal population using likelihood ratio test. 5

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