# 2019

## PHYSICS

( Major )

Paper : 5.1

*Full Marks* : 60

*Time* : 3 hours

*The figures in the margin indicate full marks for the questions*

GROUP—A

**( Mathematical Methods )**

( *Marks* : 30 )

**1.** Answer the following questions : 1×4=4

*(a)* Define a simply connected region in complex plane.

*(b)* Find out the conjugate of a complex number $7+6i$.

(c) Define a singular point of a function.

(d) Give the Euler's formula.

**2.** (a) State De Moivre's theorem. 2

(b) Find the modulus and argument of the complex number

$$\frac{1+2i}{1-(1-i)^2}$$ 2

**3.** (a) Examine whether the function $f(z) = e^z$ is an analytic function or not. 3

(b) Demonstrate a graphical representation of complex variable through Argand diagram. 2

**4.** State and prove Cauchy's integral theorem. 5





**5.** Using Cauchy's integral formula, evaluate

$$\int_C \frac{z}{(z^2-3z+2)}\,dz$$

where $C$ is the circle $|z-2| = \frac{1}{2}$. 5

**6.** (a) (i) Develop the Taylor series expansion and find the radius of convergent for $\ln z$ about $z_0 = 1$. 5

(ii) Evaluate $\oint_C \frac{dz}{z}$, where C is a circle of unit radius. 2

*Or*

(b) (i) Show that

$$\int_{-\infty}^{\infty} \frac{dx}{1+x^2} = \pi$$ 5

(ii) Give the Laurent series expansion for $f(z)$. 2

GROUP—B

**( Classical Mechanics )**

( *Marks* : 30 )

**7.** Answer the following questions/Choose the correct option : 1×4=4

*(a)* What is cyclic or ignorable coordinate?

*(b)* What is a central force?

*(c)* A particle is constrained to move along the inner surface of a fixed hemispherical bowl. The number of degrees of freedom of the particle is

*(i)* one
*(ii)* two
*(iii)* three
*(iv)* six

*(d)* For a conservative system, the potential energy does not depend upon

*(i)* force
*(ii)* generalised coordinate
*(iii)* generalised velocity
*(iv)* All of the above



( *Continued* )



**8.** Answer any *two* of the following questions : 2×2=4

*(a)* What do you understand by holonomic and non-holonomic constraints?

*(b)* Explain reduced mass in the context of two-body central force problem.

*(c)* What are generalised coordinates?

**9.** Answer any *two* of the following questions : 3×2=6

*(a)* State Kepler's laws of planetary motion.

*(b)* Show that Hamiltonian $H$ is a constant of motion if the Lagrangian $L$ is not an explicit function of time.

*(c)* Show that a two-body central force problem can be reduced to one-body problem.



( *Turn Over* )

**10.** *(a)* The Lagrangian of a problem is

$$L = \frac{1}{2}m(\dot{r}^2 + r^2\dot{\theta}^2) + V(r)$$

Identify the cyclic coordinate and the corresponding conservation law for the problem. 4

*Or*

Show that for a particle moving under a central force, the total mechanical energy of the particle is conserved.

*(b)* Use Lagrange's equations to find the equation of motion of a compound pendulum which oscillates in a vertical plane about a fixed horizontal axis. 5

Or

Establish the Hamiltonian and equations of motion of a simple pendulum.



( *Continued* )



**11.** Derive Lagrange's equation of motion from Hamilton's principle for a conservative system. 7

*Or*

Derive Lagrange's equation of motion for a conservative system using D'Alembert's principle.

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