

3 (Sem-5) PHY M 3

2019

PHYSICS

(Major)

Paper : 5.3

Full Marks : 60

Time : 3 hours

The figures in the margin indicate full marks
for the questions

GROUP—A

(Quantum Mechanics)

(Marks : 40)

1. Answer any four questions as directed : $1 \times 4 = 4$

(a) Which one of the following does not support quantum nature of radiation?

(i) Photoelectric effect

(ii) Compton effect

(iii) Interference

(iv) Black-body radiation

(Select the correct one)

(2)

- (b) The quantum mechanical form of the total energy operator of a particle moving in x -direction is

(i) $\hat{H} = \frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} + v(x)$

(ii) $\hat{H} = -\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} + v(x)$

(iii) $\hat{H} = \frac{\hbar}{2m} \frac{\partial}{\partial x} + v(x)$

(iv) $\hat{H} = -\frac{\hbar}{2m} \frac{\partial}{\partial x} + v(x)$

(Select the correct one)

- (c) State complementary principle of Neil's Bohr.

- (d) Show that the de Broglie wavelength of a particle of rest mass m_0 and kinetic energy k is given by

$$\lambda = \frac{hc}{\sqrt{k(k + 2m_0c^2)}}$$

where h = Planck's constant.

- (e) Select the correct statement :

- (i) Schrödinger equation can be derived from Newton's laws of motion.

(3)

- (ii) Schrödinger equation can be derived from Maxwell's equations of electromagnetic theory

- (iii) Schrödinger equation can be derived from both Newton's laws of motion and Maxwell's equations of electromagnetic theory.

- (iv) Schrödinger equation can not be derived from any known principle. It is a fundamental equation of quantum mechanics.

2. Answer any three questions :

2×3=6

- (a) Prove that $[x, p_x] = i\hbar$.

- (b) The threshold frequency for photoelectric emission in copper is 1.1×10^{15} Hz. Find the maximum kinetic energy of the photoelectron when light of frequency 1.5×10^{15} Hz is incident on a copper surface.

- (c) Find the phase velocity and group velocity of the de Broglie waves of an electron whose kinetic energy is 500 eV.

- (d) Normalise the wave function

$$\psi(x) = A \sin \frac{n\pi x}{L}, \quad 0 < x < L$$

where $n = 1, 2, 3, \dots$

(4)

3. Answer any two questions :

5×2=10

- (a) (i) Using uncertainty relation, show that an electron cannot reside inside a nucleus.
(ii) An excited atom gives up its excess energy by emitting a photon of characteristic frequency. The average period that elapses between the excitation of an atom and the time it radiates is 1.0×10^{-8} second. Find the inherent uncertainty in the frequency of the photon.

3+2=5

- (b) An incident X-ray photon of frequency ν_0 is scattered by a free electron at rest through an angle θ . Using relativistic expression of electron energy, show that the change in the wavelength of the photon is given by

$$\Delta\lambda = \frac{h}{m_0 c} (1 - \cos\theta)$$

where m_0 = rest mass of electron, h is the Planck's constant and c is the velocity of light. What is Compton wavelength?

4+1=5

- (c) Briefly discuss the Davisson-Germer experiment and its implications.

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(5)

4. Answer any two questions : 5×2=10

- (a) (i) Mention the properties of well-behaved wavefunctions.

- (ii) A particle limited to the x -axis has the wavefunction

$$\psi(x) = ax, \quad 0 < x < 1 \\ = 0, \quad x < 0, x > 1$$

Find the probability that the particle can be found between $x = 0.45$ and $x = 0.55$. Also, find the expectation value $\langle x \rangle$ of the particle's position. (Here a is a constant.)

2+3=5

- (b) Write one-dimensional time-dependent Schrödinger equation for a particle influenced by an arbitrary potential $V(x, t)$. Mention one important property of this equation. Starting from this equation, obtain one-dimensional time-independent Schrödinger equation. What is stationary state?

1+1+2+1=5

- (c) Derive the continuity equation from the time-dependent Schrödinger equation of a particle moving in a real potential. Give the physical interpretation of the continuity equation you derived.

4+1=5

(6)

5. Answer any two questions : $5 \times 2 = 10$

- (a) A particle of mass m is confined in a one-dimensional infinite potential well

$$V(x) = 0, \quad -a < x < a \\ = \infty, \quad |x| > a$$

Find the energy of the particle in n th quantum state.

- (b) For a linear harmonic oscillator potential $V(x) = \frac{1}{2}kx^2$, show that the Schrödinger equation (time-independent) takes the form

$$\frac{d^2\psi(q)}{dq^2} + (\lambda - q^2)\psi(q) = 0$$

where

$$\lambda = \frac{2E}{\hbar\omega}, \quad \omega = \sqrt{\frac{k}{m}}, \quad q = \alpha x, \quad \alpha = \sqrt{\frac{m\omega}{\hbar}}$$

Write the energy of the oscillator in n th quantum state. What is zero-point energy? $3+1+1=5$

- (c) One-dimensional potential barrier is defined by

$$V(x) = 0, \quad x < 0 \\ = V_0, \quad 0 < x < a \\ = 0, \quad x > a$$

Discuss the motion of a particle when $E < V_0$, where E is the energy of the

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(Continued)

(7)

particle. What is quantum mechanical tunnel effect? Give a direct experimental evidence of quantum mechanical tunnelling phenomenon. $3+1+1=5$

GROUP—B

(Astrophysics)

(Marks : 20)

6. Answer any three of the following : $2 \times 3 = 6$

- (a) Define the right ascension and declination of a celestial object.
- (b) Define local sidereal time. A star has right ascension $\alpha = 6$ h 51 m and another star has $\alpha = 10$ h 52 m. Which one of them rises earlier?
- (c) The apparent magnitude of the sun is -26.7 and that of Antares is $+1$. How much brighter does the sun appear than Antares?
- (d) What do you mean by colour index? The colour indices, B-V of four stars are (i) -0.5 , (ii) 0 , (iii) 0.5 and (iv) 1.0 . Which one of them is the hottest?
- (e) What is a parsec? Relate parsec to light year.

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(Turn Over)

7. Answer any two of the following : $4 \times 2 = 8$

(a) Draw a neat HR diagram showing the main sequence, the sun, red giant and red supergiant. Explain why the life of a massive star is shorter. $2+2=4$

(b) What is the energy-generation mechanism inside the main sequence stars? Which reaction cycle dominates in the sun like stars? Discuss carbon-nitrogen-oxygen (CNO) cycle. $1+1+2=4$

(c) Define the absolute magnitude of a star. Obtain the relation connecting absolute magnitude with apparent magnitude and distance of the star in parsec. $1+3=4$

8. Write short notes on any two of the following : $3 \times 2 = 6$

(a) Trigonometric parallax

(b) White dwarf

(c) Spectral classification of stars

(d) Celestial coordinates

The following data can be used when required :

$$e = 1.6 \times 10^{-19} \text{ eV}, c = 3 \times 10^8 \text{ m/s}$$

$$h = 6.6 \times 10^{-34} \text{ Js}, m_e = 9.1 \times 10^{-31} \text{ kg}$$

$$1 \text{ AU} = 1.49 \times 10^8 \text{ km}$$

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