Total number of printed pages–8

3 (Sem–3) PHY M1

2021

(Held in 2022)

PHYSICS

(Major)

Paper : 3·1

(Mathematical Method–III and Electrostatics)

Full Marks : 60

Time : Three hours

The figures in the margin indicate full marks for the questions.

Group–A

(Mathematical Physics)

Marks : 25

1. Answer the following questions : 1×3=3

   (a) What is the rank of a null matrix ?

   (b) Define trace of a matrix.

   (c) What do you mean by eigenvector ?

Contd.

2. Verify that $(AB)^T = B^T A^T$ where

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & -2 & 1 \end{bmatrix} \text{ and } B = \begin{bmatrix} 1 & 2 \\ 2 & 0 \\ -1 & 1 \end{bmatrix}$$ 2

3. Answer ***any two*** of the following questions : 5×2=10

(a) *(i)* If A and B are Hermitian matrices, show that $AB - BA$ is skew-Hermitian and $AB + BA$ is Hermitian. 2

(ii) Find the eigenvalues and eigenvectors of the matrix

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}.$$ 3

(b) *(i)* Find the inverse of the matrix, from the adjoint of it :

$$A = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$$ 3

(ii) If

$$A_1 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, A_2 = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}, A_3 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

show that $A_1A_2 - A_2A_1 = 2iA_2$ 2

(c) *(i)* If H is a Hermitian matrix, show that e^{iH} is unitary matrix. 2

(ii) Prove that any two eigenvectors corresponding to two distinct eigenvalues of a unitary matrix are orthogonal. 3

4. Answer ***either*** *(a), (b)* ***or*** *(c), (d)* : 5×2=10

(a) *(i)* Verify Cayley-Hamilton theorem for the matrix

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 6 \end{bmatrix}.$$ 3

(ii) If $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$, then using the value of $A^2 - 5A + 7I = 0$, find the value of A^{-1}. 2

(b) Diagonalize the following matrix : 5

$$A = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}.$$

(c) (i) Express the matrix

$$A = \begin{bmatrix} 2 & 1 & 0 \\ 1 & -1 & -2 \\ 4 & 2 & 0 \end{bmatrix}$$ as the sum of a symmetric and a skew-symmetric matrix. 2

(ii) Find the value of λ for which the matrix $A = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & \lambda \end{bmatrix}$ will be orthogonal. 3

(d) (i) Verify the theorem

$$A(adj\, A) = (adj\, A)A = |A|\, I$$

using $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix}$. 3

(ii) Show that the eigenvalues of diagonal matrix are precisely the elements in the diagonal. 2

Group–B

(Electrostatics)

Marks : 35

5. Choose the correct option : 1×4=4

(a) When a test charge is brought from infinity along the perpendicular bisector of the dipole, the work done is

(i) positive

(ii) zero

(iii) negative

(iv) None of the above

(b) The relation $D = \varepsilon E$ is true for

(i) homogeneous medium

(ii) isotropic medium

(iii) homogeneous and isotropic medium

(iv) any medium

(c) For a dipole, the electric field varies as

(i) $\frac{1}{r^2}$

(ii) $\frac{1}{r}$

(iii) $\frac{1}{r^3}$

(iv) $\frac{1}{r^4}$

(d) The unit of polarisation $\vec{P}$ is

(i) same as that of $\vec{E}$

(ii) same as that of $\vec{D}$

(iii) same as that of charge

6. Answer the following questions : 2×3=6

(a) Show that the function $\phi = 3x^2 + 8y - 3z^2$ can represent the electrostatic potential in a charge-free region.

(b) Show that $k = 1 + \chi$

where k = dielectric constant

χ = susceptibility.

(c) Find $\vec{E}$ at $(0, 0, 5)\,m$ due to $q_1 = 5\mu C$ at $(0, 3, 0)\,m$ and $q_2 = 5\mu C$ at $(3, 0, 0)\,m$.

7. Using the concept of electrical multipoles, find an expression for the electrostatic potential due to a volume distribution of charge. 5

8. Answer **any two** questions : 10×2=20

(a) (i) Write the integral form of Gauss's law in electrostatics. Using this law, determine the electric field and potential at a distance 'r' from a straight infinitely long wire having a charge λ per unit length. 1+2+2=5

(ii) Establish the boundary conditions satisfied by electric field $\vec{E}$ and electric displacement vector $\vec{D}$ at the boundary between two dielectrics. 5

(b) (i) State and prove uniqueness theorem. 4

(ii) A uniformly charged sphere of radius 'r' has the total charge Q and volume charge density ρ. Show that its electrostatic energy is

$$U = \frac{3}{5}\left(\frac{Q^2}{4\pi\varepsilon_0 r}\right).$$ 3

(iii) What is equipotential surface? What is the direction of electric field at a point on equipotential surface? 3

(c) (i) Define electrical image. With the method of electrical image, calculate the potential and the field at any point in space when a point charge is placed infront of a conducting plane of infinite extent maintained at zero potential.

1+4=5

(ii) Define $\vec{D}, \vec{E}$ and $\vec{P}$. Establish the relation $\vec{D} = \varepsilon_0 \vec{E} + \vec{P}$. 2+3=5

(d) (i) What is electric dipole ? Show that the electric field in free space due to a dipole is given by

$$\vec{E}(\vec{r}) = \frac{1}{4\pi\varepsilon_0 r^3}\left[\frac{3(\vec{P}.\vec{r})\vec{r}}{r^2} - \vec{p}\right]$$

where $\vec{p}$ is the dipole moment.

1+4=5

(ii) Establish the Clausius-Mossotti equation

$$\frac{\varepsilon_r - 1}{\varepsilon_r + 2} = \frac{N\alpha}{3\varepsilon_0}$$

for a linear dielectric material.

5
