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# 2021
## (Held in 2022)

## PHYSICS

( Honours )

Paper : PHY–HC–3016

### ( *Mathematical Physics–II* )

*Full Marks : 60*

Time : Three hours

**The figures in the margin indicate full marks for the questions.**

1. Answer the following questions : (each question carries **one** mark) $1 \times 7 = 7$

   (a) Show that $P_n(-x) = (-1)^n P_n(x)$.

   (b) $L_1(x) - L_0(x) = ?$

(c)  Express the one-dimensional heat flow equation.

(d)  $\int_0^\infty e^{-x} x^{2n-1} dx = ?$

(e)  $\beta\left(\dfrac{1}{2}, \dfrac{1}{2}\right) = ?$

(f)  Square matrix = Symmetric matrix + ?

(g)  If, $\mu^{-1} M \mu = M'$, then show that $Tr\, M = Tr\, M'$.

2.  Answer the following questions : (each question carries **2** marks)      $2\times4=8$

(a)  Show that $x = 0$ is a regular singular print for the following differential equation :

$$2x^2 \frac{d^2y}{dx^2} + 3x\frac{dy}{dx} + \left(x^2 - 4\right) y = 0$$

(b)  Can we express the one-dimensional Schrödinger's equation

$$-\frac{\hbar^2}{2m} \frac{\partial^2 \psi(x, t)}{\partial t^2} + V\psi(x, t) = i\hbar\, \frac{\partial \psi}{\partial t}(x, t)$$

in terms of space dependent and time independent equations if $V$ is a function of both $x$ and $t$? Explain.

(c)  Show that $\beta(l, m) = \beta(m, l)$.

(d)  Show that the matrix

$$\sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

is Hermitian as well as unitary.

3.  Answer **any three** questions from the following : (each question carries **5** marks)      $5\times3=15$

(a)  By the separation of variable method, solve the $t$-dependent part of the following equation :      5

$$\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$$

*Contd.*

(b) If $\begin{pmatrix} x \\ y \end{pmatrix}$ transforms to $\begin{pmatrix} x' \\ y' \end{pmatrix}$ in the way —

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$ then

show that $x'^2 + y'^2 = x^2 + y^2$.

Verify that the transformation matrix is orthogonal. 2+3=5

(c) How many real numbers are required to express a general complex matrix of dimension $2 \times 2$? Show that a $2 \times 2$ Hermitian matrix of dimension $2 \times 2$ carries four real numbers. Also, show that a skew-Hermitian matrix of dimension $2 \times 2$ carries only the real numbers. 1+2+2=5

(d) Find the Fourier's series representing $f(x) = x$, $0 < x < 2\pi$, and sketch its graph from $x = -4\pi$ to $x = +4\pi$.

3+2=5

(e) Show that

$$L'_n(x) - n\ L'_{n-1}(x) + n\ L_{n-1}(x) = 0.$$  5

4. If, $y = \sum\limits_{k=0}^{\infty} a_k\ x^{m+k}$ happens to be the power series solution of the equation,

$$2x(1-x)\frac{d^2y}{dx^2} + (1-x)\frac{dy}{dx} + 3y = 0,$$ then show

that $\quad a_{k+1} = \dfrac{-2m - 2k + 3}{2m + 2k + 1}$  10

Or

Show the following :                4+3+3=10

(1) $(n+1)\ P_{n+1} = (2n+1)\ x P_n - n\ P_{n-1}$

(2) $n\ P_n = x P'_n - P'_{n-1}$

(3) $P'_{n+1} - P'_{n-1} = (2n+1)\ P_n$

5. Solve the equation

$$\frac{\partial^2 \psi}{\partial x \, \partial t} = e^{-t} \cos x$$

given that, $\psi(t=0)=0$ and $\left. \dfrac{\partial \psi}{\partial t} \right|_{x=0} = 0$

10

**Or**

Consider a vibrating string of length $l$ fixed at both ends, given that

$$y(0, t)=0, \quad y(l, t)=0$$

$$y(x, 0) = f(x), \quad \frac{\partial y}{\partial t}(x, 0) = 0; \quad 0 < x < l$$

Solve completely the equation of vibrating string

$$\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2}.$$

10

6. If $A = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 1 \end{pmatrix}$, obtain $A^{-1}$.

From the matrix equation

$$\begin{pmatrix} 2 & 1 \\ 3 & 2 \end{pmatrix} \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} -3 & 2 \\ 5 & -3 \end{pmatrix} = \begin{pmatrix} -2 & 4 \\ 3 & -1 \end{pmatrix},$$

obtain, $a, b, c, d$.

4+6=10

**Or**

Obtain the eigenvalues and eigenvectors of the matrix

$$M = \begin{pmatrix} 2 & -i \\ i & 2 \end{pmatrix}$$

and hence diagonalize the same. 4+6=10

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