Total number of printed pages–7

3 (Sem–1) STS M1

2021

(Held in 2022)

STATISTICS

(Major)

Paper : 1·1

Full Marks : 60

Time : Three hours

The figures in the margin indicate full marks for the questions.

1. Answer the following questions (reasoning is not necessary) : 1×7=7

   (a) State the definition of mean deviation in words.

   (b) State True ***or*** False for the following relationship :

   Median = 5th decile ≠ 60th percentile

Contd.

(c) State which of the following relationships are correct :

If $x_i / f_i (i = 1, 2, ..., n)$ is a frequency distribution, then—

(i) $\sum_{i=1}^{n} f_i (x_i - \tilde{x}) = 0$

(ii) $\sum_{i=1}^{n} f_i | x_i - \bar{x} | = 0$

(d) State which of the statement is correct :

In drawing box-plot, we use

(i) range, median and standard deviation

(ii) first, second and third quartiles

(e) Mention *any two* measures of skewness.

(f) Give the definition of harmonic mean.

(g) State the advantage of coefficient of variation over standard deviation.

2. Answer the following questions : 2×4=8

(a) Write a brief note on standard deviation. 2

(b) Define first and third quartiles. 2

(c) Write a note on non-frequency data. 2

(d) Define rth raw and central moments and state the relationship between them. 1+1=2

3. Answer ***any three*** of the following : 5×3=15

(a) Prove that for any discrete distribution, standard deviation is least root-mean square deviation. 5

(b) Suppose you want to fit the equations of the types *(i)* $y = ab^x$, and *(ii)* $y = ax^b$. Explain how you would fit them by using the method of least square. 2½+2½=5

(c) Define mode and derive its formula. 1+4=5

(d) Prove that the correlation coefficient lies between zero and one. 5

(e) Write an explanatory note on Sheppard's correction for moments. 5

EITHER

4. *(a)* Compare and contrast between different measures of dispersion. 5

(b) Show that mean deviation is least when measured about median. 5

OR

5. *(a)* For the two variables X and Y, derive the line of regression of X on Y. 5

(b) Find the standard deviation of the AP series a, $a+d$, $a+2d$, ..., $a+2nd$. 5

EITHER

6. *(a)* *(i)* Define cumulants. State its properties. 1

(ii) Find the cumulant generating function of the random variable whose cumulants are

$$k_r = (r+1)!\, 2^r$$ 2

(b) Show that the variance of weighted mean

$$\frac{\sum_{i=1}^{n} w_i x_i}{\sum_{i=1}^{n} w_i}$$

of n independent random variables x_i is minimum when weights are inversely proportional to the corresponding variance. 7

OR

7. *(a)* Show that if X', Y' are the deviations of the random variables X and Y from their respective means, then

$$r = 1 - \frac{1}{2N}\sum_i \left(\frac{X_i'}{\sigma_x} - \frac{Y_i'}{\sigma_y}\right)^2$$

N is the number of observations. 4

(b) If X and Y are uncorrelated random variables with means zero and variances σ_1^2 and σ_2^2 respectively, then find σ_U^2, σ_V^2, $Cov(U, V)$ and correlation coefficient between U and V where

$$U = X\cos\alpha + Y\sin\alpha$$

$$V = X\sin\alpha - Y\cos\alpha$$ 1+1+2+2=6

EITHER

8. *(a)* In a frequency distribution, the coefficient of skewness based on the quartiles is 0·6. If the sum of the upper and lower quartiles is 100 and median is 38, find the value of quartile deviation. 4

(b) *(i)* If R is the range and σ is the standard deviation of a set of observations $x_1, x_2, \ldots, x_n$, then prove that $\sigma \le R$. 4

(ii) Write a note on factorial moments. 2

OR

9. Explain the concept of orthogonal polynomials. Let P_p the polynomial of degree p in x be given by

$$P_p = \sum_{j=0}^{p} C_{pj}\, x^j$$

where $C_{p0}, C_{p1}, \ldots, C_{pp}$ are unknown constants. Using orthogonality conditions, show that C_{pj} can be written as

$$C_{pj} = \frac{\begin{vmatrix} \mu_0 & \mu_1 & \cdots & -\mu_p & \cdots & \mu_{p-1} \\ \mu_1 & \mu_2 & \cdots & -\mu_{p+1} & \cdots & \mu_p \\ \vdots & & & & & \vdots \\ \mu_{p-1} & \mu_p & \cdots & -\mu_{2p-1} & \cdots & \mu_{2p-2} \end{vmatrix}}{\begin{vmatrix} \mu_0 & \mu_1 & \cdots & \mu_j & \cdots & \mu_{p-1} \\ \mu_1 & \mu_2 & \cdots & \mu_{j+1} & \cdots & \mu_p \\ \vdots & & & & & \vdots \\ \mu_{p-1} & \mu_p & \cdots & \mu_{j+p-1} & \cdots & \mu_{2p-1} \end{vmatrix}}$$

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