Total number of printed pages–8

3 (Sem–1) STS M 2

2021

(Held in 2022)

STATISTICS

(Major)

Paper : 1·2

(Probability–I)

Full Marks : 60

Time : Three hours

The figures in the margin indicate full marks for the questions.

1. Choose the correct option/Write true ***or*** false of the following : $1\times7=7$

   (a) Let A and B be two events. If
   $$P(A\cup B)=P(A)+P(B)$$
   then the events A and B are said to be

   (i) independent

   (ii) mutually exclusive

   (iii) mutually independent

   (iv) None of the above

Contd.

(b) Let A be an event. Then probability of the event A, i.e., $P(A) \geq 0$.

(c) If A and B are two mutually exclusive events, then $P(A/A \cup B)$ is equal to

(i) $P(A)$

(ii) $\frac{P(A)}{P(A)+P(B)}$

(iii) $\frac{P(A \cup B)}{P(A)}$

(iv) None of the above

(d) Let X be a random variable. Then the distribution function of X, i.e., $F(x)$ always satisfies the relation $F(x) \leq 1$.

(e) Let X be a random variable having probability density function $f(x)$. Then the geometric mean of the random variable is represented by the relation

$$G = \int_{-\infty}^{\infty} \log x \, f(x)\, dx$$

(f) Let X be a random variable. Then the first factorial moment about origin and the first moment about origin are same.

(g) Let X be a random variable and a be any arbitrary value. Them $M_{X-a}(t)$ is equal to

(i) $M_X(at)$

(ii) $M_{aX}(t)$

(iii) $e^{-at} M_X(t)$

(iv) None of the above

2. Answer the following questions : 2×4=8

(a) If X is a non-negative integer valued variate, then prove that

$$\sum_{k=1}^{\infty} kP(X > k) = \frac{1}{2}\left[E\left(X^2\right) - E(X)\right]$$

(b) A fair die is rolled twice. Let r be the event that the first shows a number ≤ 2 and B the event that the second throw shows at least 4. Describe the event $A \cap B$ and find $P(A \cup B)$.

(c) The distribution function F of a continuous random variable X is given by

$$\begin{aligned} F(x) &= 0 &&, x < 0 \\ &= x^2 &&, 0 \leq x \leq \tfrac{1}{2} \\ &= 1 - \frac{3(3-x)^2}{25} &&, \tfrac{1}{2} \leq x < 3 \\ &= 1 &&, x \geq 3 \end{aligned}$$

Find the p.d.f. of X with comments.

(d) From an urn containing a white and b black balls, a certain number of k balls is drawn and not replaced back, then a ball is drawn from the urn. What is the probability that this is white ball ?

3. Answer ***any three*** of the following questions : 5×3=15

(a) A bowl contains four balls, identically in all respects, numbered 1, 2, 3, 4. A ball is chosen at random. Events are defined A_1, A_2 and A_3 as follows :

Event A_i occurs iff the chosen ball is numbered either i or 4; $i = 1, 2, 3$

(i) Examine the independence of A_1, A_2 and A_3 3

(ii) Describe in words the events $(A_1 \cup A_2) \cap A_3$ 2

(b) An urn contains N balls among which W are white. A random sample of n is drawn without replacement and from this sample another random sample of size m is drawn without replacement. Find that the second sample contains exactly k white balls. 5

(c) A communication system consists of n components each of which will independently function with probability p. The total system will be able to operate effectively if at least one-half of the components function. For what value of p is a 5-component system more likely to operate effectively than a 3-component system ? 5

(d) For a random variable X, prove that

$$E\left(\frac{1}{X}\right) \geq \frac{1}{E(X)}$$ 5

(e) Concentric circles of radius 1*cm* and 3*cm* are drawn on a circle of radius 5*cm*. A man receives 10, 5 or 3 points if he hits the target inside the smaller circle, inside the middle angular region or inside the outer angular region respectively. Suppose the man hits the target with probability $\frac{1}{2}$ and then is just as likely to hit one point of the target as the other. Find the expected number E of points he scores each time he fires. 5

4. Answer ***any three*** of the following questions : 10×3=30

(a) A random variable X has distribution function

$$F(x) = 0 \quad , \quad x < 0$$
$$= \frac{1-\cos x}{2} \quad , \quad 0 \le x < \pi$$
$$= 1 \quad , \quad x > \pi$$

(i) Find the expectation of X.

(ii) Find the variance of X.

(iii) Find the median of X.

(iv) Find the mode of X.

2½×4=10

(b) Let X be a random variable with probability density function

$$f(x) = c(1-x), \; 0 < x < 1$$

Find (i) the value of c, (ii) μ_2, μ_3 and μ_4, and (iii) β_1 and β_2. 10

(c) An urn contains N balls of which M are white. A sample of n balls are drawn from the urn. Let A_k be the event that the sample contains exactly k white balls and B_j be the event that the jth ball is white. Find $P(B_j/A_k)$ when the sample is drawn (i) without replacement, and (ii) with replacement. 10

(d) Let the probability P_N that a family has n children be αp^n, $n \ge 1$ and

$$p_0 = 1 - \alpha p\left(1 + p + p^2 + \ldots\right)$$

(i) Show that for $k \ge 1$ the probability that a family contains exactly k boys is $2\alpha p^k / (2-p)^{k+1}$.

(ii) Given that a family includes at least one boy, show that the probability that there are two or more boys is $p/(2-p)$. 10

(e) Define characteristic function and describe its properties. Obtain cumulant generating functions and its properties, and hence obtain cumulants. 10

(f) (i) Define the probability—classical, relative frequency approach and axiomatic approach. Discuss their advantages and disadvantages with examples.

(ii) State and prove compound probability rules. 10
