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3 (Sem–1 /CBCS) STA HC 2

2021

(Held in 2022)

STATISTICS

(Honours)

Paper : STA–HC–1026

(Calculus)

Full Marks : 80

Time : Three hours

The figures in the margin indicate full marks for the questions.

1. Answer the following as directed : $1 \times 10 = 10$

(a) Define differential coefficient of $f(x)$ at the point $x = a$.

(b) The value of $\lim_{x \to 0} \frac{\tan x}{x}$ is

(i) 0

(ii) 1

(iii) α

(iv) None of the above

(Choose the correct option)

Contd.

(c) Evaluate $\Gamma\left(-\frac{3}{2}\right)$.

(d) State Leibnitz's theorem.

(e) Show that $\int_0^a f(x)dx = \int_0^a f(a-x)dx$

(f) Find the differential equation of lines parallel to x-axis.

(g) The integral $\beta(m,n) = \int_0^1 x^{m-1}(1-x)^{n-1}\,dx$ converges if

(i) $m > 0, n > 0$

(ii) $m < 0, n > 0$

(iii) $m > -1, n > -1$

(Choose the correct option)

(h) If $f(x,y) = 2x^2 - xy + 2y^2$, then find $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ at the point $(1,2)$.

(i) The differential equation

$$\left(\frac{d^2y}{dx^2}\right)^2 - 2\left(\frac{dy}{dx}\right)^2 + 5y = 0 \quad \text{is}$$

(i) an ordinary differential equation

(ii) of order two and degree two

(iii) called partial differential equation

(Choose the incorrect option)

(j) Find the value of

$$\lim_{x \to \alpha} \frac{x^4}{e^x}$$

2. Answer the following questions : 2×5=10

(a) Examine the differentiability at $x=0$ of the function f defined on the set of real number as follows :

$$f(x) = x^2 \sin\frac{1}{x}, \text{ if } x \neq 0$$
$$= 0, \text{ if } x = 0$$

(b) Evaluate $\lim_{x \to 0} (\sin x \log x)$

(c) Show that $f(x) = x^3 - 6x^2 + 24x + 1$ has neither a maximum nor a minimum.

Contd.

(d) Obtain a differential equation from the relation

$$y = A \sin x + B \cos x + x \sin x$$

(e) Show that for $l > 0,\ m > 0$

$$\int_a^b (x-a)^{l-1}(b-x)^{m-1}\,dx = (b-a)^{l+m-1}\,\beta(l,m)$$

3. Answer **any four** from the following questions : 5×4=20

(a) Show that if a function is differentiable at a point, then it is continuous at that point but the converse is not necessarily true.

(b) Show that the necessary and sufficient condition for the differential equation $Mdx + Ndy = 0$ to be be exact is

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

(c) Evaluate $\int_1^{\log 8}\int_1^{\log y} e^{x+y}\,dy\,dx$

(d) If (a,b) be a point of the domain of definition of a function f such that

(i) f_x is continuous at (a,b)

(ii) f_y exists at (a,b), then show f is differentiable at (a,b).

(e) If $u = \sin^{-1}\left(\frac{x^3+y^3}{x+y}\right)$, then using Euler's theorem show that

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 2\tan u$$

(f) Prove that $\beta(m,n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$

4. (a) (i) If $y = \sin^{-1}x$, then using Leibnitz's theorem prove that

$$\left(1-x^2\right)y_{n+2} - (2n+1)x\,y_{n+1} - n^2 y_n = 0$$

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(ii) Test the continuity and differentiability of the function

$$f(x)=\begin{cases}1+x & \text{if} \quad x\le 2\\ 5-x & \text{if} \quad x\ge 2\end{cases}$$ 4

at $x=2$

Or

(b) Solve the differential equation

$$\frac{dy}{dx}=\frac{x+2y-3}{2x+y-3}$$ 10

5. (a) (i) For a positive number P, show that

$$\Gamma(P)\Gamma\left(P+\frac{1}{2}\right)2^{2P-1}=\sqrt{\pi}\,\Gamma(2P)$$ 6

(ii) Evaluate $\lim\limits_{x\to 0}(\cos x)^{\frac{1}{x^2}}$ 4

Or

(b) (i) Evaluate $\int\limits_0^{\pi/2}\log \sin x\, dx$ 5

(ii) If $u=2(ax+by)^2-(x^2+y^2)$ and $a^2+b^2=1$, find the value of

$$\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}.$$ 5

6. (a) (i) Show that the function $u=x^3+y^3-3ay$ has a maximum or minimum at the point (a,a) according as a is negative or positive. 5

(ii) If $f(x,y)=\dfrac{xy(x^2-y^2)}{x^2+y^2}$;

$(x,y)\neq(0,0)$, $f(0,0)=0$, then show that at the origin $f_{xy}\neq f_{yx}$. 5

Or

(b) (i) Solve the differential equation :

$$\frac{d^2y}{dx^2}-4\frac{dy}{dx}+5y=\sin x$$ 5

(ii) Define Clairaut's equation. Explain the general solution of Clairaut's equation. 5

7. (a) (i) If $u^3+v^3=x+y$,

$u^2+v^2=x^3+y^3$, prove that

$$\frac{\partial(u,v)}{\partial(x,y)}=\frac{y^2-x^2}{2uv(u-v)}$$ 5

(ii) Solve the partial differential equation : 5

$$\left(\frac{y^2z}{x}\right)P+xzq=y^2$$

Or

(b) If f is defined and continuous on the rectangle $R=[a,b;c,d]$, and if

(i) $f_x(x,y)$ exists and is continuous on the rectangle R, and

(ii) $g(x)=\int_c^d f(x,y)dy$ for $x\in[a,b]$

then show that g is differentiable on $[a,b]$ and $g'(x)=\int_c^d f_x(x,y)dy$

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