2018

COMPUTER SCIENCE

(Major)

Paper : 2.2

(Discrete Mathematics)

Full Marks : 60

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. Answer the following as directed : 1×7=7

   (a) When is a relation said to be partial order relation?

   (b) Can we construct a simple graph with five vertices having degrees 2, 3, 3, 3 and 5?

   (c) What is contradiction?

   (d) A graph with all vertices having equal degree is known as _____.

   (Fill in the blank)

   (e) What is the minimum rank of a non-zero matrix?

(f) How many words can be formed using the letter A thrice, the letter B twice and the letter C once?

(g) State the absorption law in Boolean algebra.

2. Answer any *four* of the following : 2×4=8

(a) Prove that every invertible matrix has a unique inverse.

(b) For two non-empty sets A and B, prove that $A \subseteq B$ iff $B' \subseteq A'$.

(c) Construct a truth table for the following proposition :

$$(q \to p) \wedge (\sim p \wedge q)$$

(d) Define the term 'predicates'.

(e) Define a bipartite graph. Draw a complete bipartite graph symbolized by $K_{2,3}$.

(f) If ${}^nP_r = {}^nP_{r+1}$ and ${}^nC_r = {}^nC_{r-1}$, find n and r.

3. Answer any *three* of the following : 5×3=15

(a) From a class of 12 boys and 10 girls, 10 students to be chosen for a competition at least including 4 boys and 4 girls. The two girls who won the prizes last year should be included. In how many ways can the selection be made?

(b) Prove that every tree has either one or two centres.

(c) When is a statement formula said to be tautology? Show that the formula $q \vee (p \wedge \sim q) \vee (\sim p \wedge \sim q)$ is a tautology.

(d) Consider a function $f: R_+ \to [-5, \infty]$ given by $f(x) = 9x^2 + 6x - 5$. Show that f is one-one and onto (R_+ is the set of all non-negative real numbers).

(e) State and prove Cayley-Hamilton theorem.

4. Answer any *three* of the following : 10×3=30

(a) (i) Using the principle of mathematical induction, prove that for any positive integer n, $6^n - 1$ is divisible by 5.

(ii) On the set Z of all integers, consider a relation $R = \{(a, b) : a, b \in Z; a - b$ is divisible by $3\}$. Show that R is an equivalence relation in Z. Also find the partition of Z into mutually disjoint equivalent classes.

(b) (i) Find the eigenvalue for the following matrix :

$$\begin{pmatrix} 5 & 2 & 1 \\ 2 & 5 & 1 \\ 2 & 1 & 5 \end{pmatrix}$$

(ii) For what values of λ and μ, do the equations $x+2y+3z=6$, $x+3y+5z=9$ and $2x+5y+\lambda z=\mu$ have no solution, unique solution and infinite number of solutions?

(c) (i) When is a statement formula said to be in normal form? With the help of truth table, obtain the principal disjunctive normal form of the formula $(\sim p \rightarrow r) \wedge (q \leftrightarrow p)$.

(ii) Translate the following into predicate calculus formula :

(1) Some real numbers are integers.

(2) Some mathematicians are not good in Chemistry.

(3) All integers are either even integers or odd integers.

(4) No resident of Assam is a resident of America.

(5) There are Web programmers who know PERL but not PHP.

(d) (i) Prove that

$$^{n+1}C_r + ^{n+1}C_{r+1} = ^{n+2}C_{r+1},\ n \geq r$$

(ii) State and prove the pigeonhole principle. State generalized pigeon-hole principle.

(e) (i) If L denotes a lattice, then prove that for $a, b, c \in L$, $a \vee a = a$ and $a \vee (a \wedge b) = a$.

(ii) Define rank of a matrix. Find the rank of the following matrix :

$$\begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 3 & 5 & 4 \end{pmatrix}$$

(f) (i) Define the following in a graph (any *two*) :

. Degree of a vertex, Centre of a tree, Binary tree, Adjacency matrix

(ii) Write an algorithm on breadth first search technique of graph traversal.

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