Total number of printed pages–12

3 (Sem–3) PHY M 1

2022

PHYSICS

(Major)

Paper : 3·1

(Mathematical Methods–III and Electrostatics)

Full Marks : 60

Time : Three hours

The figures in the margin indicate full marks for the questions.

GROUP–A

(Mathematical Physics)

(Marks : 25)

1. Answer the following questions : $1\times3=3$

   (a) In matrices, find the value of $(A+B+C)^2$.

   (b) Show that $(A^2)^{-1} = (A^{-1})^2$.

(c) What is the rank of a zero matrix?

2. Check whether

$$\begin{pmatrix} i/2 & \sqrt{3}/2 \\ \sqrt{3}/2 & i/2 \end{pmatrix}$$

is a unitary matrix. 2

3. Answer **any two** of the following questions : 5×2=10

(a) (i) For an orthogonal matrix, if λ is an eigenvalue, what is the other value? 1

(ii) If

$$A_\alpha = \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix}, \quad A_\beta = \begin{pmatrix} \cos\beta & \sin\beta \\ -\sin\beta & \cos\beta \end{pmatrix}$$

check whether $A_\alpha A_\beta = A_{\alpha+\beta}$ is correct or not. 2

(iii) If

$$\begin{array}{cc} & \begin{array}{cc} \textit{Room I} & \textit{Room II} \end{array} \\ A = & \begin{pmatrix} 10 & 12 \\ 9 & 14 \\ 15 & 14 \end{pmatrix} \begin{array}{l} \textit{Flat I} \\ \textit{Flat II} \\ \textit{Flat III} \end{array} \end{array}$$

gives the power consumed in two rooms within three flats and

$$X = \begin{pmatrix} 10 \\ 5 \end{pmatrix} \begin{array}{l} \textit{Room I} \\ \textit{Room II} \end{array}$$

gives the number of electrical items in rooms, then what information does $Y = AX$ yield and where is its highest value? 2

(b) (i) If

$$A = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}, \text{ then } A^n = \begin{pmatrix} \cos n\theta & -\sin n\theta \\ \sin n\theta & \cos n\theta \end{pmatrix}$$

What does this result mean geometrically? 1

(ii) If A and B are Hermitian matrices, show that $AB - BA$ is skew-Hermitian whereas $AB + BA$ is Hermitian. 2

(iii) Compute the adjoint of a matrix

$$A = \begin{pmatrix} 0 & 3 & 2 \\ -1 & 2 & 5 \\ 5 & 0 & 3 \end{pmatrix}$$ 2

(c) (i) Derive the expression for the force F' acting on a body in a constant rotating frame in terms of applied force F and two other fictitious forces. Name the fictitious forces. 3+1=4

(ii) What is the effect of diurnal rotation of the earth on the acceleration due to gravity of earth at a place where latitude is λ? 1

4. Answer **either** (a), (b) **or** (c), (d) : 5×2=10

(a) (i) If $A = \begin{pmatrix} 1 & \alpha \\ 0 & 1 \end{pmatrix}$

then find the value of A^n. 2

(ii) In an electrical network

$$I_1 - I_2 + I_3 = 0$$
$$2I_2 - 3I_3 = 0$$
$$5I_1 + 3I_2 = 2$$

Find the currents by matrix method. 3

(b) (i) If

$$A = \begin{pmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & a & 1 \end{pmatrix}, \quad A^{-1} = \begin{pmatrix} \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \\ -4 & 3 & c \\ \frac{5}{2} & -\frac{3}{2} & \frac{1}{2} \end{pmatrix}$$

then find the value of $a + c$. 3

(ii) If

$$A = \begin{pmatrix} 2 & -3 \\ 0 & 4 \end{pmatrix}, \quad B = \begin{pmatrix} 5 & 2 \\ 2 & 1 \end{pmatrix}$$

find $A - B$ and also a symmetric matrix out of it. 2

(c) (i) Verify Cayley-Hamilton theorem for the matrix

$$A = \begin{pmatrix} 1 & 1 & 0 \\ 3 & 0 & 1 \\ 2 & 3 & 1 \end{pmatrix}$$ 3

(ii) If $A = \begin{pmatrix} 3 & 1 \\ -1 & 2 \end{pmatrix}$

then using the value of

$$A^2 - 5A + 7I = 0$$

find the value of A^{-1}. 2

(d) (i) Given

$$x_1 = 3y_1 + 2y_2$$
$$x_2 = -y_1 + 4y_2$$

Find the transformation equation for y_1, y_2 by matrix method. 3

(ii) If

$$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

satisfies the equation

$$x^2 - (a+d)\,x + k = 0$$

then find the relation among k, a, b, c, d 2

GROUP–B

(Electrostatics)

(Marks : 35)

5. Choose the correct option : 1×3=3

(a) The relation $D = \varepsilon E$ is true for

(i) any medium

(ii) homogeneous medium

(iii) isotropic medium

(iv) homogeneous and isotropic medium

(b) Uniqueness of electric field strength E means

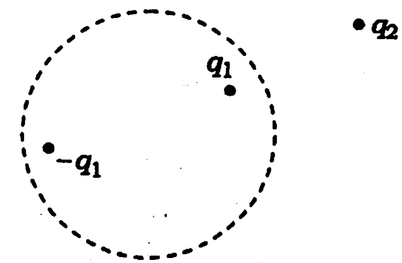
(i) $V_1 = V_2$

(ii) $\nabla V_1 = \nabla V_2$

(iii) $V_1 = V_2 + \text{constant}$

(iv) Both (ii) and (iii)

(c) A Gaussian surface in the figure below is shown by the dotted line :



The electric field on the surface will be

(i) due to q_1, q_2 only

(ii) due to q_2 only

(iii) due to all

(iv) zero

6. Answer the following questions : 3×2=6

(a) Given an electric field in a limited region surrounding an origin in the X, Y plane for which the potential is represented by

$$\phi = ax^2 + C$$

where a, C are positive constants. Find the components of the field intensity. Where is the potential extremum? Where is the field intensity a minimum? 1+1+1=3

Or

Show that $K = 1 + \chi$

where K = dielectric constant

χ = susceptibility

Define polarization charges. 2+1=3

(b) Check which of the two expressions below for electrical potential is applicable for a charged region. Correspondingly find the charge density : 2+1=3

(i) $3x^2 + y^2 + 2z^2$

(ii) $x^2 - y^2 + 8z$

7. Answer **either** (a) **or** (b) : 6

(a) (i) Find the electric field at a point located at a distance r_1 from the axis of a dipole of length d. Show that if $d/r_1 << 1$, the field at that behaves as $E = 2p/r_1^3$, p = dipole moment. 3

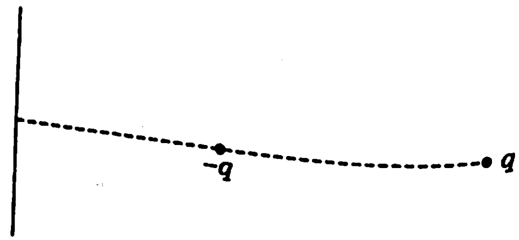
(ii) Define equipotential surface. What is the direction of electric field at a point on equipotential surface? 3

(b) (i) A sphere of radius b is uniformly charged by charge density ρ. Calculate the electrostatic energy of the sphere. 3

(ii) Show that the divergence of electric field of a point charge vanishes. 3

8. Answer ***any two*** questions : 10×2=20

(a) (i) An electric dipole of length $2\,mm$ having charge of value $q = 2.0 \times 10^{-8} C$ is placed near a long line charge of density $4.0 \times 10^{-4} cm^{-1}$ as shown in the figure



such that the negative charge is at a distance of $20\,m$ from the line charge, the force acting on the dipole is $0{\cdot}11k$ Newton. Find k. 5

(ii) Establish the boundary conditions satisfied by electric field E and electric displacement vector D at the boundary between two dielectrics. 5

(b) (i) Using Laplace's equation, show that the electric field is constant in the region between the two parallel plates and it is toward the plate of lower potential. 5

(ii) There is a solid sphere of radius R having volume charge density

$$\rho = \rho_0 (1 - r/R)$$

where ρ_0 is constant and r is the distance from the centre of the sphere. Find the electric intensity E at a point inside the sphere using Gauss' law. 5

(c) (i) Draw the field lines of $\vec{E}, \vec{P}, \vec{D}$ in the region between the plates of a capacitor (of thickness d) with the dielectric (of thickness t) in between the plates (given $d > t$). Show that $\vec{D} = \varepsilon_0 \vec{E} + \vec{P}$. 3+2=5

(ii) Deduce the relation between dielectric constant of a fluid and its polarizability. 5

(d) (i) Define electrical image. Find the value of surface density of the induced charge on an infinite conducting plane due to a point charge. Draw the necessary figure. State the region where Laplace's equation is satisfied in such a case. 1+6+1=8

(ii) An electron is at a distance of $10\mathring{A}$ from an infinite plane conductor. Calculate the force experienced by the proton. 2