3 (Sem–3/CBCS) STA HC 3

2022

STATISTICS

(Honours)

Paper : STA–HC–3036

(Mathematical Analysis)

Full Marks : 60

Time : Three hours

The figures in the margin indicate full marks for the questions.

1. Answer ***any seven*** from the following questions : $1 \times 7 = 7$

   (a) Identify the wrong statement :

   (i) The set R of real numbers is the neighbourhood of each of its points.

   (ii) The set Q of rationals is the neighbourhood of each of its points.

   (iii) The open interval $]a, b[$ is the neighbourhood of each of its points.

   (Choose the correct option)

(b) The set $\left\{\frac{1}{n} : n \in N\right\}$ has only one limit point, zero, which is not a member of the set. *(State True **or** False)*

(c) If $\sum u_n$ is a positive term series, such that $\lim_{n \to \infty} \frac{u_n + 1}{u_n} = l$, then the series converges if

(i) $l < 1$

(ii) $l > 1$

(iii) $l = 1$

(iv) $l = 0$ *(Choose the correct option)*

(d) The positive term geometric series $1 + r + r^2 + \ldots..$ converges for $r < 1$ and diverges for $r \geq 1$.

*(State True **or** False)*

(e) Define alternating series.

(f) A function which is continuous in a closed interval is also uniformly continuous in that interval.

*(State True **or** False)*

(g) State the geometrical interpretation of Lagrange's mean value theorem.

(h) State the expansion of *cos x*.

(i) The *n*th divided difference can be expressed as the product of multiple integrals. *(State True **or** False)*

(j) Define the operators μ, δ used in calculus of finite differences.

(k) State Stirling's formula for factorial *n*, when *n* is large.

(l) Which of the following is not correct ?

(i) Simpson's rule gives a better result than the trapezoidal rule.

(ii) Weddle's rule is generally more accurate than any of the others.

(iii) Simpson's $\frac{1}{3}$ rule is better than Simpon's $\frac{3}{8}$ rule

(iv) None of the above

2. Answer ***any four*** from the following questions : 2×4=8

(a) Show that the series

$$\frac{1}{1^P}-\frac{1}{2^P}+\frac{1}{3^P}-\frac{1}{4^P}+\ldots$$ converges for $P>0$.

(b) Define bounded and unbounded sets. Is the set of natural numbers bounded ?

(c) If M and N are neighbourhood of a point x, then prove that $M \cap N$ is also a neighbourhood of x.

(d) State Taylor's theorem with Lagrange's and Cauchy's form of remainder.

(e) Using Lagrange's mean value theorem prove that

$$|\tan^{-1}x-\tan^{-1}y| \le |x-y| \; \forall x, y \in R$$

(f) Solve the difference equation

$$u_{x+1}-au_x=0, a \neq 1$$

(g) Write a note on numerical integration.

(h) Find the first three divided differences of the function $\frac{1}{x}$ for the arguments a, b, c, d.

3. Answer ***any three*** from the following questions : 5×3=15

(a) Show that $\lim_{n\to\infty} n^{\frac{1}{n}}=1$

(b) Expand e^x by Maclaurin's infinite series.

(c) (i) Define limit superior and limit inferior of a bounded sequence. 2

(ii) Prove that the intersection of a finite family of open sets is open. 3

(d) Show that between any two roots of $e^x \cos x=1$, there exists at least one root of $e^x \sin x-1=0$.

(e) State the following :

(i) d'Alembert's ratio text 2

(ii) Raabe's test 2

(iii) Absolute convergence of series 1

(f) State and prove Simpson's $\frac{1}{3}$ rule.

(g) Establish the relation between operator E of calculus of finite differences and differential coefficient D of differential calculus.

Also show that

$$\nabla = 1 - e^{-hD}$$

where ∇ is called backward difference operation. 3+2=5

(h) State and prove Gauss's forward interpolation formula.

4. Answer ***any three*** of the following questions : 10×3=30

(a) State and prove Cauchy's general principle of convergence.

(b) Prove that if $f(x)$ is a function, which is

(i) continuous in the closed interval $[a, b]$

(ii) differentiable in the open interval (a, b) and

(iii) $f(a) = f(b)$, then there exists one value of x say $\xi \in]a, b[$ such that $f(\xi) = 0$

Also give the geometrical meaning of Rolle's theorem. 8+2=10

(c) Expand $log(1+x)$ by Maclaurin's infinite series.

(d) (i) State Cauchy's first theorem on limits. 1

(ii) Show that

$$\lim_{n \to \infty} \left[\frac{1}{\sqrt{n^2+1}} + \frac{1}{\sqrt{n^2+2}} + ... + \frac{1}{\sqrt{n^2+n}} \right] = 1$$

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(iii) Define (i) monotonic sequence and (ii) derived sets. 3

(e) (i) State and prove Lagrange's mean value theorem. Also give its geometrical interpretation. 5+2=7

(ii) Show that a necessary condition for convergence of an infinite series $\sum u_n$ is that $\lim_{n \to \infty} \sum u_n = 0$. 3

(f) State and prove Lagrange's interpolation formula for unequal intervals. Also show that the sum of the Lagrangian coefficients is unity. 7+3=10

(g) (i) State and prove Simpson's $\frac{3}{8}$ rule. Also state its assumptions. 6

(ii) Solve the difference equation $u_{x+1} - 3^x u_x = 0$ 4

(h) (i) Write a note on use of various interpolation formulae. 5

(ii) Evaluate

$$\Delta^2 \left[\frac{5x+12}{x^2+5x+6} \right], \text{ taking } h = 1$$ 5