Total number of printed pages–11

3 (Sem–4/CBCS) STA HC 1

2022

STATISTICS

(Honours)

Paper STA–HC–4016

(Statistical Inference)

Full Marks : 60

Time : Three hours

The figures in the margin indicate full marks for the questions.

1. Answer the following questions as directed : ***(any seven)*** $1\times7=7$

   (a) If T is an unbiased estimator for θ, then T^2 is a biased estimator for θ^2. *(State true **or** false)*

   (b) Estimators by the method of moments are not in general consistent and efficient. *(State true **or** false)*

Contd.

(c) Type I error is accepting H_0, when H_0 is false. *(State true **or** false)*

(d) In sampling from a $N(\mu, \sigma^2)$ population, the sample mean is a/an ________ estimator of μ.

(Fill up the blank)

(e) Let $x_1, x_2, \dots x_n$ be the sample observations constitute a space called the

(i) critical region

(ii) sample space

(iii) Both (i) and (ii)

(iv) None of the above

(Choose the correct option)

(f) Area of critical region depends on

(i) level of significance

(ii) size of type II error

(iii) calculated value of the test statistic.

(iv) None of the above

(Choose the correct option)

(g) Neyman–Pearson lemma provides

(i) an unbiased test

(ii) a most powerful test

(iii) an admissible test

(iv) None of the above

(Choose the correct option)

(h) In 1933, the theory of testing of hypothesis was profounded by ________. *(Fill up the blank)*

(i) Maximum likelihood estimators are always consistent estimators but need not be unbiased.

*(State true **or** false)*

(j) If of the two consistent estimators T_1, T_2 of a certain parameter θ, we have $V(T_1) < V(T_2)$ for all n then for all sample sizes

(i) T_1 is more consistent than T_2

(ii) T_2 is more consistent than T_1

(iii) T_1 is more efficient than T_2

(iv) T_2 is more efficient than T_1

(Choose the correct option)

2. Answer the following questions : ***(any four)*** 2×4=8

(a) Describe best critical region for a test.

(b) Find the maximum likelihood estimator (MLE) of θ for the following probability distribution :

$$f(x,\theta) = \theta e^{-\theta x},\ x > 0,\ \theta > 0$$

(c) Write short notes on simple and composite hypotheses.

(d) If $x_1, x_2, \ldots x_n$ is a random sample from a normal population $N(\mu, 1)$, then show that $t = \frac{1}{n}\sum_{i=1}^{n} x_i^2$ is an unbiased estimator of $\mu^2 + 1$.

(e) Prove that in sampling from a $N(\mu, \sigma^2)$ population, the sample mean is a consistent estimator of μ.

(f) Write down the condition for the Cramer-Rao lower bound for the variance of the estimator to be attained.

(g) State two asymptotic properties of likelihood ratio (LR) test.

(h) Differentiate between estimator and estimate.

3. Answer the following questions : ***(any three)*** 5×3=15

(a) If $X_1, X_2, X_3, \ldots X_n$ are random observations on a Bernoulli variate X taking the value 1 with probability P and the value O with probability $(1-P)$, show that

$$\frac{\sum x_i}{n}\left(1 - \frac{\sum x_i}{n}\right) \text{ is a consistent}$$

estimator of $P(1-P)$.

(b) State and prove the invariance property of consistent estimator.

(c) Define minimum variance unbiased estimator (MVUE). If T_1 is an MVUE for θ and T_2 is any other unbiased estimator of θ with efficiency e, then show that no linear combination of T_1 and T_2 is an MVUE.

(d) Obtain the minimum variance bound (MVB) estimator for μ in normal population $N(\mu, \sigma^2)$, where σ^2 is known.

(e) State the regularity conditions for Cramer-Rao inequality.

(f) State and prove Neyman-Pearson lemma.

(g) State Neyman's factorization theorem. Let $(X_1, X_2, \ldots X_n)$ denote a random sample from a distribution with p.d.f. $f(x,\theta) = \theta x^{\theta-1}, \quad 0 < x < 1, \theta > 0$
$= 0 \quad , \text{ otherwise}$
Show that the product $(X_1, X_2, \ldots X_n)$ is a sufficient statistic for θ.

(h) Examine whether a best critical region exists for testing the null hypothesis $H_0 : \theta = \theta_0$ against the alternative hypothesis $H_1 : \theta > \theta_0$ for the parameter θ of the distribution

$$f(x,\theta) = \frac{1+\theta}{(x+\theta)^2}, 1 \le x \le \alpha$$

4. Answer **any three** of the following : 10×3=30

(a) (i) If $\{T_n\}$ be a sequence of estimators such that for all $\theta \in \Theta$, $E_\theta(T_n) \to r(\theta), n \to \alpha$ and $Var\theta(T_n) \to 0$, as $n \to \alpha$, then 'T_n' is a consistent estimator of $r(\theta)$. 7

(ii) Write a note on Blackwellization process. 3

(b) (i) Explain the concept of unbiasedness and efficiency. A random sample $(X_1, X_2, X_3, X_4, X_5)$ of size 5 is drawn from a normal population with unknown mean μ. Consider the following estimators to estimate μ:

$$T_1 = \frac{X_1 + X_2 + X_3 + X_4 + X_5}{5}$$

$$T_2 = \frac{X_1 + X_2}{2} + X_3$$

$$T_3 = \frac{2X_1 + X_2 + \lambda X_3}{3}$$

where λ is such that T_3 is an unbiased estimator of μ.

(1) Find λ

(2) Are T_1 and T_2 unbiased estimators ?

(3) Which is the best estimator among T_1, T_2 and T_3 ?

1+1+1+2+2=7

(ii) If $x \geq 1$ is the critical region for testing $H_0 : \theta = 2$ against the alternative $\theta = 1$, on the basis of the single observation from the population

$f(x, \theta) = \theta e^{-\theta x}$; $0 \leq x \leq \infty$, obtain the values of type I error. 3

(c) Given the probability density function $f(x : \theta) = \left[\pi\left\{1 + (x - \theta)^2\right\}\right]^{-1}$; $-\alpha < x < \alpha$, $-\alpha < \theta < \alpha$; show that the Cramer-Rao lower bound of variance of an unbiased estimator of θ is $2/n$, where n is the size of the random sample from this distribution.

(d) Show that for normal distribution with zero mean and variance σ^2, the best critical region for $H_0 : \sigma = \sigma_0$ against the alternative $H_1 : \sigma = \sigma_1$ is of the form

$$\sum_{i=1}^{n} x_i^2 \leq a_\alpha \text{ for } \sigma_0 > \sigma_1 \text{ and}$$

$$\sum_{i=1}^{n} x_i^2 \geq b_\alpha \text{ for } \sigma_0 > \sigma_1$$

Contd.

(e) Explain the likelihood ratio test. Let $x_1, x_2, \ldots x_n$ be a random sample from $N(\mu, \sigma^2)$, where σ^2 is known. Develop likelihood ratio test for $H_0: \mu = \mu_0$ against $H_1: \mu > \mu_0$.

(f) Use the Neyman–Pearson lemma to obtain the critical region for testing $\theta = \theta_0$ against $\theta = \theta_1$ for $\theta_1 > \theta_0$ and $\theta_1 < \theta_0$. Show that there exists no uniformly most powerful (UMP) test for testing $\theta = \theta_0$ against $\theta_1 \neq \theta_0$ for the following pdf :

$$f(x, \theta) = \theta e^{-\theta x}, \; x > 0$$

(g) What is meant by a statistical hypothesis ? Explain the concepts of type I and type II errors. Show that a most powerful test is necessarily unbiased.

1+(2+2)+5=10

(h) Write short notes on : ***(any two)*** 5×2=10

(i) Method of minimum χ^2

(ii) Uniformly most powerful test

(iii) Sequential probability ratio test (SPRT)

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