Total number of printed pages–31

3 (Sem–5/CBCS) MAT HE 4/5/6

2022

MATHEMATICS

(Honours Elective)

Answer the Questions from any one Option.

OPTION–A

Paper : MAT–HE–5046

(Linear Programming)

DSE(H)–2

Full Marks : 80

Time : Three hours

OPTION–B

Paper : MAT–HE–5056

(Spherical Trigonometry and Astronomy)

Full Marks : 80

Time : Three hours

OPTION–C

Paper : MAT–HE–5066

(Programming in C)

Full Marks : 60

Time : Three hours

The figures in the margin indicate full marks for the questions.

Contd.

OPTION–A

Paper : MAT–HE–5046

(Linear Programming)

DSE (H)–2

Full Marks : 80

Time : Three hours

The figures in the margin indicate full marks for the questions.

1. Answer ***any ten*** questions from the following : *(Choose the correct answer)* $1 \times 10 = 10$

(i) A function $f(x)$ is said to the strictly convex at x if for two other distinct points x_1 and x_2

(a) $f[\lambda x_1 + (1-\lambda)x_2] < \lambda f(x_1) + (1-\lambda) f(x_2)$, where $0 \le \lambda \le 1$

(b) $f[\lambda x_1 + (1-\lambda)x_2] < \lambda f(x_1) + (1-\lambda) f(x_2)$, where $0 < \lambda < 1$

(c) $f[\lambda x_1 + (1-\lambda)x_2] \le \lambda f(x_1) + (1-\lambda) f(x_2)$, where $0 \le \lambda \le 1$

(d) None of the above

(ii) If X is the set of eight vertices of a cube, then the convex hull $C(X)$ is the

(a) surface of the cube

(b) vertices of the cubic

(c) whole cube

(d) None of the above

(iii) The extreme points of the convex set of feasible solutions are

(a) finite in number

(b) infinite in number

(c) either finite or infinite

(d) None of the above

(iv) In a linear programming problem

$$\max Z = cx$$

subject to $Ax \ge b,\ x \ge 0$, c is called

(a) coefficient vector

(b) column vector

(c) price vector

(d) None of the above

(v) Consider a system $Ax = b$ of m equations in n unknowns, $n > m$. Then maximum number of basic solution is

(a) ${}^{m}C_{n}$

(b) ${}^{n}C_{m-1}$

(c) ${}^{n}C_{n-m}$

(d) None of the above

(vi) A basic feasible solution of an LPP is said to be non-degenerate BFS if

(a) none of the basic variable zero

(b) at least one of the basic variable zero

(c) exactly one of the basic variable zero

(d) None of the above

(vii) If the LPP $max\ Z = cx$ such that $Ax = b, x \geq 0$ has a feasible solution then at least one of the BFS will be

(a) maximal

(b) minimal

(c) optimal

(d) None of the above

(viii) If for any basic feasible solution of an LPP, there is some column α_j in A but not in B for $c_j - z_j > 0$ and $y_{ij} \leq 0\ (i = 1, 2, \cdots, m)$, then the problem has an unbounded solution if the objective function is to be

(a) maximized

(b) minimized

(c) *either* maximized *or* minimized

(d) None of the above

(ix) Standared form of LPP is

(a) Min $Z = cx$ s.t. $Ax \geq b,\ x \geq 0$

(b) Max $Z = cx$ s.t. $Ax \leq b,\ x \geq 0$

(c) Max $Z = cx$ s.t. $Ax \geq b,\ x \geq 0$

(d) None of the above

(x) The incoming vector in a simplex table will be taken as α_k if

(a) $\Delta_k = max\ \Delta_j$

(b) $\Delta_j = max\ \Delta_k$

(c) entries of α_k are all negative

(d) None of the above

(xi) If we consider dual of an LPP, then in the dual the requirement vector of the primal problem becomes

(a) objective function

(b) price vector

(c) variable

(d) None of the above

(xii) The necessary and sufficient condition for any LPP and its dual to have optimal solution is that

(a) both have basic solution

(b) both have unbounded solution

(c) both have feasible solution

(d) None of the above

(xiii) If any of the constraint in the primal is perfect equality, the corresponding dual variable is

(a) perfect equality

(b) unrestricted in sign

(c) strictly inequality

(d) None of the above

(xiv) In an assignment problem

$$min\, Z = \sum_{i=1}^{n}\sum_{j=1}^{n} c_{ij} x_{ij} ,$$

$$\sum_{j=1}^{n} x_{ij} = 1, \quad i = 1, 2, \cdots, n$$ means that

(a) only one job is done by i-th person $i = 1, 2, \cdots, n$

(b) i-th person is assigned to j-th job

(c) only one person should be assigned to the j-th job, $j = 1, 2, \cdots, n$

(d) None of the above

(xv) In a transportation problem if we apply North-West Corner method, we always get

(a) non-degenerated BFS

(b) degenerated BFS

(c) optimal solution

(d) None of the above

(xvi) For optimality test in a transportation problem, number of allocation in independent position must be

(a) $m+n$

(b) $m+n+1$

(c) $m+n-1$

(d) None of the above

(xvii) In a transportation table for cell evaluation we use the formula

(a) $c_{rs} = u_r + v_s,\ d_{ij} = (u_i + v_j) - c_{ij}$

(b) $c_{rs} = u_r + v_s,\ d_{ij} = c_{ij} - (u_i + v_j)$

(c) $c_{rs} = u_r + v_s,\ d_{ij} = u_i + v_j$

(d) None of the above

(xviii) Define finite game a "Game Theory".

2. Answer ***any five*** from the following: 2×5=10

(a) Show that the FS $x_1 = 1,\ x_2 = 0,\ x_3 = 1$ and $z = 6$ to the system of equations

$$x_1 + x_2 + x_3 = 2$$

$$x_1 - x_2 + x_3 = 2,\ x_j \geq 0$$ which minimize

$$z = 2x_1 + 3x_2 + 4x_3$$ is not basic.

(b) Is $x_1 = 1,\ x_2 = \frac{1}{2},\ x_3 = x_4 = x_5 = 0$ a basic solution to the following system?

$$x_1 + 2x_2 + x_3 + x_4 = 2$$

$$x_1 + 2x_2 + \frac{1}{2}x_3 + x_5 = 2$$

(c) Examine convexity of the set

$$S = \left\{(x_1, x_2) : x_1^2 + x_2^2 \leq 4\right\}$$

(d) Define artificial variable. Give an example.

(e) Define unbounded solution of an LPP. How can we determine that the solution of an LPP is unbounded?

(f) What is two phase method to solve an LPP? Mention the phases?

(g) Write "complementary slackness theorem" of a dual problem.

(h) How can we find entering vector in a simplex table?

(i) Write the 'Test of optimality' for primal dual method.

(j) What is cost matrix of an assignment problem?

Contd.

3. Answer **any four** from the following : 5×4=20

(a) Prove that the set of all feasible solutions of an LPP is a convex set. [Assume that the set is non empty]

(b) If the objective function of an LPP assume its optimal value at more than one extreme point, then prove that every convex combination of these extreme points gives the optimal value of the objective function.

(c) Give the dual of the following LPP :

$$Max\ Z = 2x_1 + 3x_2 + x_3$$

$$\text{s.t. } 4x_1 + 3x_2 + x_3 = 6$$

$$x_1 + 2x_2 + 5x_3 = 4$$

$$x_1, x_2, x_3 \geq 0$$

(d) Solve the following transportation problem by North-West Corner method

	S_1	S_2	S_3	S_4	a_i
O_1	1	2	1	4	30
O_2	3	3	2	1	50
O_3	4	2	5	9	20
b_j	20	40	30	40	100

(e) Solve the game whose pay-off matrix is

$$\begin{bmatrix} -1 & -2 & 8 \\ 7 & 5 & -1 \\ 6 & 0 & 12 \end{bmatrix}$$

(f) Mark the feasible region represented by the constraint conditions

$$x_1 + x_2 \leq 1,\ 3x_1 + x_2 \geq 3,\ x_1 \geq 0,\ x_2 \geq 0$$

(g) Find initial BFS of the following LPP :

$$Max\ Z = 2x_1 + 3x_2$$

$$\text{s.t. } -x_1 + 2x_2 \leq 4$$

$$x_1 + x_2 \leq 6$$

$$x_1 + 3x_2 \leq 9$$

$$x_1, x_2 \text{ unrestricted}$$

(h) If in an assignment problem, a constant is added or substracted to every element of row (or column) of the cost matrix $[c_{ij}]$, then prove that an assignment which minimizes the total cost for one matrix, also minimize the total cost for the other matrix.

4. Answer ***any four*** from the following : 10×4=40

(a) A soft drink plant has two bottling machines *A* and *B*. It produces and sells 8 ounce and 16 ounce bottles. The following data is available

Machine	8 ounce	16 ounce
A	100/minute	40/minute
B	60/minute	75/minute

The machines can run 8 hours per day, 5 days per week. Weekly production of drink cannot exceed 3,00,000 ounces and the market can absorb 25,000 eight ounce bottles and 7,000 sixteen ounce bottles per week, Profit on these bottle is 15 paise and 25 paise per bottle respectively. The planner wishes to minimize his profit subject to all the production and marketing restrictions. Formulate it as an LPP and solve graphically.

(b) State and prove fundamental theorem of LPP.

(c) Using simplex alogorithm solve the problem

$$Max\ Z = 2x_1 + 5x_2 + 7x_3$$

$$\text{s.t. } 3x_1 + 2x_2 + 4x_3 \le 100$$
$$x_1 + 4x_2 + 2x_2 \le 100$$
$$x_1 + x_2 + 3x_3 \le 100$$
$$x_1, x_2, x_3 \ge 0$$

(d) Use dual to solve the LPP

$$Min\ Z = 2x_1 + x_2$$

$$\text{s.t. } 3x_1 + x_2 \ge 3$$
$$4x_1 + 3x_2 \ge 6$$
$$x_1 + 2x_2 \ge 3$$
$$x_1, x_2 \ge 0$$

(e) Solve the following transportation problem :

Market

Plant	*A*	*B*	*C*	*D*	Available
X	10	22	10	20	8
Y	15	20	12	8	13
Z	20	12	10	15	11
Required	5	11	8	8	32

(f) State and prove Fundamental Duality theorem.

(g) The pay-off matrix for A in a two persons zero sum game is given below. Determine the value of the game and the optimum strategies for both players

		B: I	B: II	B: III
A	I	−1	2	1
	II	1	−2	2
	III	3	4	−3

(h) Find an optimal solution of the following LPP without using simplex method :

$$Max\ Z = 2x_1 + 3x_2 + 4x_3 + 7x_4$$

$$\text{s.t.}\quad 2x_1 + 3x_2 - x_3 + 4x_4 = 8$$

$$x_1 - 2x_2 + 6x_3 - 7x_4 = -3$$

$$x_i \geq 0,\ i = 1, 2, 3, 4$$

(i) If an LPP

$$Max\ Z = cx \quad \text{s.t.}\quad Ax = b,\ x \geq 0$$

where A is $mX(m+n)$ matrix of co-efficients given by $A = (\alpha_1, \alpha_2, \cdots, \alpha_{m+n})$, has at least one feasible solution, then prove that it has at least one basic feasible solution.

(j) A company has four territories open and four salesman available for assignment. The territories are not equally rich in their sales potential; it is estimated that a typical salesman operating in each territory would bring in the following annual sales

Territories	I	II	III	IV
Annual sales (*Rs.*)	60,000	50,000	40,000	30,000

The four salesman are also considered to differ in ability; it is estimated that, working under the same conditions, their yearly sales would be proportionally as follows :

Salesman :	A	B	C	D
Proportion :	7	5	5	4

If the criterion is maximum expected total sales, the intuitive answer is to assign the best salesman to the richest territory, the next best salesman to the second richest and so on. Verify this answer by the assignment technique.

———