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3 (Sem-2/CBCS) STA HC 2

2023

STATISTICS

(Honours Core)

Paper : STA-HC-2026

(Algebra)

Full Marks : 60

Time : Three hours

The figures in the margin, indicate full marks for the questions.

1. Answer the following questions as directed :

1×7=7

(a) If two rows of a determinant are identical, the value of the determinant is _____. (Fill in the blank)

(b) If A be an $n \times n$ matrix with the property that $A^2 - 3A + 2I = 0$, where I is the $n \times n$ identity matrix; which of the following statements are true ?

(i) $\text{Det } (A - I) = 0$

Contd.

- (ii) A is invertible
- (iii) A has n distinct eigenvalues
- (iv) All of the above

(Choose the correct answer)

- (c) If the roots of the quadratic equation $x^2 - (k+2)x + 121 = 0$ are equal, then positive value of k is :

- (i) 20
- (ii) 21
- (iii) 24
- (iv) None of the above

(Choose the correct answer)

- (d) If V_1 and V_2 are 3-dimensional subspaces of a 4-dimensional vector space V , then the smallest possible dimension of $V_1 \cap V_2$ is _____.

(Fill in the blank)

- (e) For a non-singular matrix A , $(A')^{-1} = (A^{-1})'$. (State True or False)

- (f) If A and B are symmetric matrices of same order, then $(AB' - BA')$ is a

- (i) skew symmetric matrix
- (ii) null matrix

- (iii) symmetric matrix

- (iv) None of the above

(Choose the correct option)

- (g) For which value of x will the matrix given below become singular ?

$$\begin{bmatrix} 8 & x & 0 \\ 4 & 0 & 2 \\ 12 & 6 & 0 \end{bmatrix}$$

- (i) 4
- (ii) 6
- (iii) 8
- (iv) 12

(Choose the correct option)

2. Answer the following questions : $2 \times 4 = 8$

- (a) Define sub-space of an n -vector V_n .

- (b) Solve the equation

$$3x^3 - 16x^2 + 23x - 6 = 0$$

if the product of two roots is 1.

- (c) If A and B are two square matrices of order n , then show that $\text{trace}(AB) = \text{trace}(BA)$.

(d) If A and B are n -rowed orthogonal matrices, then show that AB is also orthogonal matrix.

3. Answer **any three** of the following questions : $5 \times 3 = 15$

(a) Solve the equation .

$$\begin{vmatrix} 3x-8 & 3 & 3 \\ 3 & 3x-8 & 3 \\ 3 & 3 & 3x-8 \end{vmatrix} = 0$$

(b) If A be an $n \times n$ matrix, prove that $|\text{Adj } A| = |A|^{n-1}$

(c) Prove that the system of equation $AX = B$ is consistent, if and only if the coefficient matrix A and the augmented matrix $[A, B]$ are of the same rank.

(d) State and prove Cayley-Hamilton theorem.

(e) If α, β and γ are the roots of the equation $x^3 + px^2 + qx + r = 0$, find the values of

(i) $(\alpha + \beta)(\beta + \gamma)(\gamma + \alpha)$

(ii) $\sum \alpha^2 \beta^2$

(iii) $\sum (\alpha - \beta)^2$

4. Answer the following questions : $10 \times 3 = 30$

(a) Write down in matrix form of the system of equations

$$2x - y + 3z = 9$$

$$x + y + z = 6$$

$$x - y + z = 2$$

and find A^{-1} , if

$$A = \begin{bmatrix} 2 & -1 & 3 \\ 1 & 1 & 1 \\ 1 & -1 & 1 \end{bmatrix}$$

and hence solve the given equation.

10

Or

(b) (i) Show that if A is an orthogonal matrix, then A' and A^{-1} are also orthogonal. 5

(ii) Verify the orthogonality of the matrix

$$\frac{1}{3} \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ 2 & 2 & -1 \end{bmatrix}$$

5

- (c) Find matrices P and Q so that PAQ is of the normal form, where

$$A = \begin{bmatrix} 1 & 0 & -2 \\ 2 & 3 & -4 \\ 3 & 3 & -6 \end{bmatrix}$$

Also find the rank of A .

10

Or

- (d) (i) Write the process of finding the eigenvalues and eigenvectors of a matrix. 3
(ii) Determine the eigenvalues and the corresponding eigenvectors of the matrix 7

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & 3 \\ 0 & 0 & 2 \end{bmatrix}$$

- (e) (i) Explain the solution of cubic equations by Cardan's method. 6
(ii) If α, β, γ are the roots of the equation $x^3 + 2x^2 + 3x + 4 = 0$, find the equation whose roots are $1 + \frac{1}{\alpha}, 1 + \frac{1}{\beta}, 1 + \frac{1}{\gamma}$ 4

Or

- (f) (i) Define vector spaces. 2
(ii) Show that the intersection of *any* two subspaces of a vector space is again a subspace of the space. 8