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3 (Sem–1/CBCS) MAT HC 2

2023

MATHEMATICS

(Honours Core)

Paper : MAT–HC–1026

(Algebra)

Full Marks : 80

Time : Three hours

The figures in the margin indicate full marks for the questions.

1. Answer the following questions : 1×10=10

   (a) Find the polar representation of $-i$.

   (b) Write the n^{th} roots of unity.

   (c) State De Moivre's theorem.

   (d) Define a statement.

   (e) Draw the truth table for the statement formula $\sim(\sim p \wedge q)$.

Contd.

(f) Define the composite mapping $(g \circ f): R \to R$, where f and g are defined as $f: R \to R$ such that $f(x) = \sin x, \forall x \in R$ and $g: R \to R$ such that $g(x) = x^2, \forall x \in R$.

(g) Define a universal relation in a set.

(h) Write the greatest common divisor of *two* relatively prime integers.

(i) Fill in the blank :

If two rows of an $h \times n$ matrix A are interchanged to produced B, then det B = ______.

(j) Given, $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 5 \\ 3 \end{bmatrix}$.

Compute $B^T A^T$.

2. Answer the following questions : 2×5=10

(a) Find the principal value of argument of $-2-2i$.

(b) Construct a truth table for the statement formula :

$(p \to q) \to (\sim q \to \sim p)$.

(c) Give an example of a relation which is reflexive, but is *neither* symmetric *nor* transitive.

(d) Find the adjoint of the matrix

$$A = \begin{bmatrix} 1 & 2 \\ 3 & -5 \end{bmatrix}$$

(e) Evaluate the determinant by using row reduction to Echelon form

$$\begin{vmatrix} 1 & 5 & -6 \\ -1 & -4 & 4 \\ -2 & -7 & 9 \end{vmatrix}$$

3. Answer ***any four*** questions : 5×4=20

(a) If $a = \cos\alpha + i\sin\alpha$, $b = \cos\beta + i\sin\beta$, $c = \cos r + i\sin r$ and $a + b + c = 0$, then prove that $\frac{1}{a} + \frac{1}{b} + \frac{1}{c} = 0$

(b) Prove that the power set of a set with n elements has 2^n elements. Write down the power set of $s = \{a\}$.

(c) Prove that two equivalence classes are *either* disjoint *or* identical.

(d) Solve the system of equations :

$$\begin{aligned} x + 3y - 2z &= 0 \\ 2x - y + 4z &= 0 \\ x - 11y + 14z &= 0 \end{aligned}$$

(e) Verify that the adjoint of a diagonal matrix of order 3 is a diagonal matrix.

(f) Use Cramer's rule to compute the solutions to the system :

$$\begin{aligned} 2x_1 + x_2 &= 7 \\ -3x_1 + x_3 &= -8 \\ x_2 + 2x_3 &= -3 \end{aligned}$$

4. Answer **either** (a) **or** (b) of the following questions : 10×4=40

(a) (i) Prove that the amplitude of a purely imaginary number is $\frac{\pi}{2}$ *or* $-\frac{\pi}{2}$ according as the number is positive *or* negative. 5

(ii) Prove that

$$(1+\sin\theta+i\cos\theta)^n + (1+\sin\theta-i\cos\theta)^n = 2^{n+1}\cos^n\left(\frac{\pi}{4}-\frac{\theta}{2}\right)\cos\left(\frac{n\pi}{4}-\frac{n\theta}{2}\right)$$

5

(b) (i) If $(a_1+ib_1)(a_2+ib_2)\dots(a_n+ib_n) = A+iB$, prove that

$$(a_1^2+b_1^2)(a_2^2+b_2^2)\dots(a_n^2+b_n^2) = A^2+B^2$$

5

(ii) If a function $f : A \to B$ is one-one onto then prove that the inverse function $f^{-1} : B \to A$ is also one-one onto. 5

5. (a) (i) For any three sets A, B, C. Show that $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$. 5

(ii) If A, B, C are three sets such that $A \cup C = B \cup C$ and $A \cap C = B \cap C$, then prove that A = B. 5

(b) (i) State the division algorithm. Also find the gcd (720, 150). 2+3=5

(ii) Prove that $7^n - 1$ is divisible by 6 for all integers $n \geq 0$. 5

6. (a) (i) Let m be a positive integer. Then prove that the congruence classes $[a]$ and $[b]$ for all $a, b \in Z$, satisfy *either* $[a] \cap [b] = \phi$ *or* $[a] = [b]$. 5

(ii) If A is a non-singular matrix, then show that adjadj $A = |A|^{n-2} A$. 5

(b) (i) Reduce the following matrix to Echelon form and hence find its rank :

$$\begin{bmatrix} 1 & 3 & 4 & 3 \\ 3 & 9 & 12 & 9 \\ 1 & 3 & 4 & 1 \end{bmatrix}$$ 5

(ii) Investigate for what values of a and b the following system of equations have no solutions

$$\begin{aligned} x + y + z &= 6 \\ x + 2y + 3z &= 10 \\ x + 2y + 2z &= b \end{aligned}$$ 5

7. (a) (i) If the vectors u, v, w are linearly independent, then show that the vectors $u + v$, $u - v$, $u - 2v + w$ are also linearly independent. 5

(ii) When the inverse of a square matrix exist? Find the inverse of the following matrix.

$$A = \begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix}$$ 5

(b) Prove that the system of equations $AX = B$ is consistent if and only if the coefficient matrix A and the augmented matrix $[A\ B]$ are of the same rank. 10
