3 (Sem–3/CBCS) STA HC 1

2023

STATISTICS

(Honours Core)

Paper : STA–HC–3016

(Sampling Distributions)

Full Marks : 60

Time : Three hours

The figures in the margin indicate full marks for the questions.

1. Answer the following questions as directed : 1×7=7

(a) The standard error of the sampling distribution of the statistic $(\bar{x})$ is *(Choose the correct option)*

(i) $\sigma^2\sqrt{2/n}$

(ii) $\sigma/\sqrt{n}$

(iii) $\sqrt{\sigma^2/2n}$

(iv) None of the above

Contd.

(b) The cumulative distribution function of the largest order statistic $X_{(n)}$ is given by _____. *(Fill in the blank)*

(c) For large n if $X \sim N(nP, nPQ)$, then

$Z = \frac{X - nP}{\sqrt{nPQ}}$ follows

(Choose the correct option)

(i) $N(0, \sigma^2)$

(ii) $N(\mu, \sigma^2)$

(iii) $N(0, 1)$

(iv) $N(\mu, \sigma)$

(d) If $X_i, i = 1, 2, \ldots n$ are n independent normal variates with mean (μ_i) and SD (σ_i), then $\sum_{i=1}^{n} \left(\frac{X_i - \mu}{\sigma}\right)^2$ is a chi-square variate with n d.f.

*(Write true **or** false)*

(e) The probability of type I error is called _____. *(Fill in the blank)*

(f) State the pdf of Fisher's t-distribution.

(g) Equality of two population variances can be tested by

(Choose the correct option)

(i) t-test

(ii) F-test

(iii) Both *(i)* and *(ii)*

(iv) None of the above

2. Answer the following questions : 2×4=8

(a) Derive the cumulative distribution function of $X_{(1)}$.

(b) Write *any two* applications of chi-square statistic.

(c) Explain one tailed and two tailed tests.

(d) Write the assumptions for students t-test.

3. Answer ***any three*** questions from the following : 5×3=15

(a) Explain in brief the test used for testing the difference between two proportions for large samples.

(b) Find the joint distribution of r^{th} and s^{th} order statistics $(r < s)$ in taking random sample from a continuous distribution.

(c) Derive cumulant generating function (c.g.f.) of chi-square distribution. Also find its mean and variance using c.g.f.

(d) Define F statistic. Write down the p.d.f. of Snedecor's F distribution. Derive the mode of F distribution. 1+1+3=5

(e) In $F(n_1, n_2)$ distribution and if $n_2 \to \infty$, then prove that $n_1 F$ follows chi-square distribution with n_1 d.f.

Answer ***either*** 4. *(a)* ***or*** 4. *(b)* :

4. (a) Obtain the distribution of sample median in case of order statistics. 10

(b) (i) Let X_1 and X_2 be two independent normal variates with the same normal distribution $N(\mu, \sigma^2)$. Obtain the distribution of

$$Y = \frac{X_1 + X_2 - 2\mu}{\sqrt{|X_1 - X_2|^2}}.$$ 5

(ii) If X is t-distributed with K degrees of freedom, show that $\frac{1}{1+(X^2/K)}$ has a beta distribution. 5

Answer ***either*** 5. *(a)* ***or*** 5. *(b)* :

5. *(a)* If X_1 and X_2 are two independent chi-square variate with n_1 and n_2 d.f. respectively, then show that X_1/X_2 is a $\beta_2(n_1/2, n_2/2)$ variate. 10

(b) *(i)* Describe the steps in detail for testing a statistical hypothesis. 5

(ii) For t-distribution with n d.f., derive the mean deviation about mean. 5

Answer ***either*** 6. *(a)* ***or*** 6. *(b)* :

6. *(a)* *(i)* Derive the probability density function of student's t. 7

(ii) Comment on the graph of t-distribution. 3

(b) Write three applications of F distribution.

Let X_1 and X_2 be a random sample of size 2 from $N(0, 1)$ and Y_1 and Y_2 be a random sample of size 2 from $N(1, 1)$ and let Y_i's be independent of X_i's. Find the distribution of the following : 3+7=10

(i) $\dfrac{(X_1 + X_2)^2}{(X_2 - X_1)^2}$

(ii) $\dfrac{(Y_1 + Y_2 - 2)^2}{(X_2 - X_1)^2}$
