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3 (Sem–3/CBCS) STA HC 3

2023

STATISTICS

(Honours Core)

Paper : STA–HC–3036

(Mathematical Analysis)

Full Marks : 60

Time : Three hours

The figures in the margin indicate full marks for the questions.

1. Answer the following questions as directed : $1 \times 7 = 7$

(a) The least upper bound of the set $\left\{ \frac{1}{n}, n \in N \right\}$ is

(i) 1

(ii) 0

(iii) –1

(iv) None of the above

(Pick up the correct option)

(b) Identify the wrong statement :

(i) The intersection of two open sets is open.

(ii) Every open set is an union of open intervals.

(iii) The union of two open sets is closed.

(iv) The set of all integers is countable.

(c) State Bolzano-Weierstrass theorem.

(d) A sequence cannot converge to more than one limit. *(State True or False)*

(e) The value of $\Delta^4 (1-x)^4$, the interval of differencing being unity is

(i) 0

(ii) 1

(iii) 4

(iv) 24

(Choose the correct option)

(f) Given the following data :

Income per day not exceeding (Rs.) :	10	18	20	28	40
Workers :	12	32	68	80	100

To interpolate number of workers for income not exceeding Rs.30 per day, the suitable method is :

(i) Newton's backward formula

(ii) Lagrange's formula

(iii) Binomial expansion method

(iv) Gauss backward formula

(Choose the correct option)

(g) If the n^{th} differences of a tabulated function $f(x)$ are constant, the value of independent variables are taken at equal intervals, then

(i) $f(x)$ is a polynomial of degree n

(ii) $f(x)$ is constant

(iii) $f(x)$ is zero

(iv) $f(x)$ is a polynomial of degree $(n-1)$

(Choose the correct option)

2. Answer the following questions : 2×4=8

(a) Using Lagrange's mean value theorem, prove that

$$\left|\tan^{-1}x-\tan^{-1}y\right|\leq\left|x-y\right|\ \forall\ x, y\in R$$

(b) Prove that every convergent sequence is bounded.

(c) Show that for any real number x,

$$\lim\frac{x^n}{n!}=0.$$

(d) State Taylor's theorem with Lagrange's and Cauchy's form of remainder.

3. Answer ***any three*** of the following questions : 5×3=15

(a) State and prove Cauchy's first theorem on limits.

(b) Expand $\sin x$ by Maclaurin's infinite series.

(c) State and prove Rolle's theorem.

(d) If four equidistant values u_{-1}, u_0, u_1 and u_2 are given and a value u_x is interpolated by Lagrange's formula, show that

$$u_x = yu_0 + xu_1 + \frac{y(y^2-1)}{3!}\Delta^2 u_{-1} + \frac{x(x^2-1)}{3!}\Delta^2 u_0$$

where $x+y=1$.

(e) Show that the n^{th} order divided difference of a polynomial of n^{th} degree is constant.

4. Answer *(a)* ***or*** *(b)* of the following questions :

(a) (i) Evaluate $\lim\limits_{x\to 0}\dfrac{x-\tan x}{x^3}$ 3

(ii) Expand $(1+x)^n$ by Maclaurin's infinite series. 7

(b) (i) Prove that a function which is uniformly continuous on an interval is continuous on that interval. 4

(ii) Let $f(x)=\begin{cases} x^p \sin\left(\frac{1}{x}\right) & , \ x \neq 0 \\ 0 & , \ x = 0 \end{cases}$

Obtain p such that (i) $f(x)$ is continuous at $x=0$ (ii) $f(x)$ is differentiable at $x=0$. 6

5. Answer *(a)* **or** *(b)* of the following questions :

(a) State and prove Cauchy's general principle of convergence. 10

(b) (i) Solve the difference equation :

$$u_{x+2} - 4u_x = 9x^2$$ 4

(ii) Write a note on use of various interpolation formulae. 6

6. Answer *(a)* **or** *(b)*:

(a) (i) State Cauchy's n^{th} root test and Leibnitz's test for the convergence of alternating series. 4

(ii) Show that

$$\lim_{n\to\infty}\left[\frac{1}{\sqrt{n^2+1}}+\frac{1}{\sqrt{n^2+2}}+.....+\frac{1}{\sqrt{n^2+n}}\right]=1$$

6

(b) (i) Derive Gauss's interpolation formula for central differences. 5

(ii) State and prove Weddle's rule for numerical integration. 5