

2018

MATHEMATICS

(Major)

Paper : 2.2

(Differential Equation)

Full Marks : 80

Time : 3 hours

*The figures in the margin indicate full marks
for the questions*

1. Answer the following as directed : $1 \times 10 = 10$

(a) Write down the degree of the differential equation

$$y = \sqrt{x} \frac{dy}{dx} + \frac{k}{\frac{dy}{dx}}$$

(b) Is the differential equation

$$\frac{dy}{dx} = \frac{y}{x} + \sin \frac{y}{x}$$

homogeneous?

(c) What is the integrating factor of the differential equation $x dy - y dx = 0$?

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(Turn Over)

- (d) Write the particular integral of the differential equation

$$(D^2 - 3D + 2)y = e^{5x}$$

- (e) What do you mean by trajectory of a given family of curves?

- (f) Write down the general solution of the differential equation

$$y = px + e^p ; \quad p = \frac{dy}{dx}$$

- (g) Write the conditions for exactness of a total differential equation

$$P dx + Q dy + R dz = 0$$

- (h) The partial differential equations can be formed by the elimination of

- (i) arbitrary constants only
- (ii) arbitrary functions only
- (iii) arbitrary functions and arbitrary constants
- (iv) None of the above

(Choose the correct answer)

- (i) Write the standard form of the linear partial differential equation of order one.

- (i) Write down Lagrange's auxiliary equations of the linear partial differential equation

$$x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} + z \frac{\partial u}{\partial z} = xyz$$

2. Answer the following questions : 2×5=10

- (a) Find the differential equation of all straight lines passing through the origin.

- (b) Solve :

$$\cos(x+y) dy = dx$$

- (c) Solve :

$$\frac{d^3 y}{dx^3} - 2 \frac{d^2 y}{dx^2} + 4 \frac{dy}{dx} - 8y = 0$$

- (d) Solve :

$$yz dx + 2zx dy - 3xy dz = 0$$

- (e) Construct the partial differential equation by eliminating a and b from

$$z = ax + (1-a)y + b$$

3. Answer any four parts : 5×4=20

- (a) Solve :

$$x^2 \left(\frac{dy}{dx} \right)^2 + xy \frac{dy}{dx} - 6y^2 = 0$$

(b) Show that the system of confocal and coaxial parabolas $y^2 = 4a(x+a)$ is self-orthogonal, a being parameter.

(c) Solve :

$$(D^2 - 2D + 4)y = e^x \cos x$$

(d) Solve :

$$x^3 \frac{d^3 y}{dx^3} + 2x^2 \frac{d^2 y}{dx^2} + 2y = 10 \left(x + \frac{1}{x} \right)$$

(e) Apply variation of parameters to solve the differential equation

$$\frac{d^2 y}{dx^2} + 4y = 4 \tan 2x$$

(f) Solve :

$$\frac{dx}{dt} - 7x + y = 0; \quad \frac{dy}{dt} - 2x - 5y = 0$$

4. Answer either (a) and (b) or (c) and (d) : 5+5=10

(a) Show that the differential equation

$$\frac{dy}{dx} + \frac{ax + hy + g}{hx + by + f} = 0$$

is exact and hence solve it.

- (b) Write down Bernoulli's differential equation. Solve the following differential equation by reducing it in linear form :

$$\frac{dy}{dx} + xy = xy^2$$

- (c) Reduce the differential equation

$$(y + xp)^2 = x^2 p$$

where $p = \frac{dy}{dx}$ to Clairaut's form by substituting $xy = v$ and hence solve the equation.

- (d) Solve

$$\frac{d^2 y}{dx^2} + y = 0$$

given $y = 2$ for $x = 0$; $y = -2$ for $x = \frac{\pi}{2}$.

5. Answer either (a) and (b) or (c) and (d) : 5+5=10

- (a) Solve

$$\sin^2 x \left(\frac{d^2 y}{dx^2} \right) = 2y$$

given $y = \cot x$ is a solution.

- (b) Find $f(z)$ such that the equation
- $$\left(\frac{y^2 + z^2 - x^2}{2x} \right) dx - y dy + f(z) dz = 0$$
- is integrable. Hence solve it.

- (c) Solve :

$$x \frac{d^2 y}{dx^2} - (2x - 1) \frac{dy}{dx} + (x - 1)y = 0$$

- (d) Solve :

$$\frac{dx}{x(y^2 - z^2)} = \frac{dy}{y(z^2 - x^2)} = \frac{dz}{z(x^2 - y^2)}$$

6. Answer either (a) and (b) or (c) and (d) : 5+5=10

- (a) Solve :

$$xz^3 dx - z dy + 2y dz = 0$$

- (b) Solve

$$x^4 \frac{d^2 y}{dx^2} + 2x^3 \frac{dy}{dx} + x^2 y = 0$$

by changing the independent variable x to z .

- (c) Reduce the differential equation

$$\frac{d^2 y}{dx^2} - \frac{2}{x} \frac{dy}{dx} + \left(1 + \frac{2}{x^2} \right) y = x e^x$$

to its normal form and hence solve it.

- (d) Derive the partial differential equation by elimination of arbitrary function ϕ from the equation $\phi(u, v) = 0$, where u and v are functions of x, y and z .

7. Answer either (a) and (b) or (c) and (d) : $5+5=10$

- (a) Solve by Lagrange method :

$$z(xp - yq) = y^2 - x^2$$

- (b) Find the integral surface of the partial differential equation

$$(x - y)p + (y - x - z)q = z$$

through the circle $z = 1, x^2 + y^2 = 1$.

- (c) Solve by Charpit's method :

$$(p^2 + q^2)y = qz$$

- (d) Find the complete integral of

$$q^2 = z^2 p^2 (1 - p^2)$$

Find also the singular integral, if it exists.

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