3 (Sem–6/CBCS) STA HE 3

2024

STATISTICS

(Honours Elective)

Paper : STA–HE–6036

(Actuarial Statistics)

Full Marks : 60

Time : Three hours

The figures in the margin indicate full marks for the questions.

1. Answer the following questions as directed : $1 \times 7 = 7$

   (a) What is reinsurance arrangement ?

   (b) In a life table, state the relationship between L_x and T_x.

   (c) Define force of mortality.

(d) If $S(x)$ is a survival function of x, then what is the value of $S(0)$?

(e) In the individual risk model, the number of claims is assumed to be random variable. *(State True **or** False)*

(f) Define curtate future lifetime.

(g) What is utility function ?

2. Answer ***all*** the questions briefly : 2×4=8

(a) Find the moment generating function of Poisson distribution with parameter λ .

(b) State different types of utility function.

(c) Show that a logarithmic utility function is risk averse.

(d) State the *two* conditions that must hold for a risk to be insurable.

3. Answer ***any three*** questions from the following : 5×3=15

(a) A random variable X has a beta distribution with parameters $\alpha > 0$ and $\beta > 0$ with its pdf

$$f(x) = \frac{\sqrt{(\alpha+\beta)}}{\sqrt{\alpha}\sqrt{\beta}} x^{\alpha-1}(1-x)^{\beta-1}, \quad 0 < x < 1$$

Show that $E\left[X^n\right] = \frac{\sqrt{(\alpha+\beta)}\sqrt{(n+\alpha)}}{\sqrt{\alpha}\sqrt{(n+\alpha+\beta)}}$

and hence find the mean and variance of X.

(b) Write a brief note on properties of premium principles.

(c) Show that the maximum premium P that an individual with utility function $U(x) = -exp[-\beta x]$ is prepared to pay for complete insurance cover against a random loss, X, where $X \sim N(\mu, \sigma^2)$, is an increasing function of β, and explain this result.

(d) Obtain $E(Z)$, where Z is not single premium in terms of V for

(i) n year term insurance ;

(ii) whole life insurance.

(e) "The minimum acceptable premium that an insurer will like to charge is larger than the expected loss." Establish the validity of the above statement.

4. Answer ***any three*** : 10×3=30

(a) (i) What is deterministic survivorship group ? State the characteristics of deterministic survivorship group. 1+3=4

(ii) State the procedure how you would find the median future lifetime of x. 3

(iii) Derive the expression for the average number of years lived between x and $x+n$ by the lx survivors at age x. 3

(b) Assume that each of 100 independent lives

— is age x,

— is subject to a constant force of mortality, $\mu = 0{\cdot}04$, and

— is insured for a death benefit amount of 10 units, payable at the moment of death.

The death benefit payments are to be withdrawn from an investment fund earning $\delta = 0{\cdot}06$. Calculate the minimum amount at $t = 0$ so that the probability is approximately 0·95 that sufficient funds will be on hand to withdraw the benefit payment at the death of each individual.

(c) (i) Write a detailed note on insurance and utility theory. 5

(ii) A decision maker's utility function is given by $u(w) = -e^{-5w}$. The decision maker has two random economic prospects (gains) available. The outcome of the first, denoted by X, is distributed as $N(5, 2)$. The second prospect, denoted by Y, is distributed as $N(6, 2{\cdot}5)$. Which prospect will be preferred and why? 5

(d) Derive the form of the individual risk model, specifying clearly the meaning of any notations, you use.

(e) Derive the expressions of life table functions using survival functions.

(f) The random variables X_1, X_2 and X_3 are independent with distributions defined by columns of the following table :

x	$f_1(x)$	$f_2(x)$	$f_3(x)$
0	0·4	0·5	0·6
1	0·3	0·2	0·0
2	0·2	0·1	0·1
3	0·1	0·1	0·1
4	–	0·1	0·1
5	–	–	0·1

Derive the probability mass function and cumulative distribution function of $S + X_1 + X_2 + X_3$.

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