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STATISTICS

(Major)

Paper : 4.2

(Descriptive Statistics—II & Probability—II)

Full Marks : 60

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. Answer the following as directed : 1×7=7

(a) A sequence of random variables $X_1, X_2, \ldots, X_n$ is said to converge in probability to a constant a, if for any $\varepsilon > 0$, $\lim_{n \to \infty} P\{|X_n - a| \geq \varepsilon\} = 1$.

(State True or False)

(b) If $\phi(t)$ is characteristic function of the variate X, then $\phi(0) =$ _____ .

(Fill in the blank)

(c) If the distribution function of an r.v. is symmetrical about zero, then $\phi_X(t)$ is real valued and even function of t.

(State True or False)

(d) A random variable X has a mean value of 5 and variance of 3. What is the least value of $P\{|X - 5| < 6\}$?

(e) Define uniqueness theorem of characteristic function.

(f) Standard error plays a very important role in the large sample theory and forms the basis of the testing of hypothesis.

(State True or False)

(g) If the parameter space of a Markov process is _____, then the Markov process is called a Markov chain.

(Fill in the blank)

2. Answer the following questions in short : 2×4=8

(a) Write the importance of characteristic function.

(b) What is Markov process?

(c) State the Bernoulli's laws of large number.

(d) Write the transition problem in matrix form.

3. Answer any *three* of the following questions : 5×3=15

(a) Define clearly the Chapman–Kolmogorov theorem and Chapman-Kolmogorov equation.

(b) The sex ratio of birth is sometimes given by the ratio of male to female births instead of the proportion of male to the total births. If Z is the ratio, i.e., $Z = \frac{p}{q}$, show that the standard error of Z is $\frac{1}{1+Z}\sqrt{Z/n}$ approximately, n being large.

(c) State and prove the Tchebysheff's inequality.

(d) Show that every stochastic process with independent increment is a Markov process.

(e) Examine whether the weak law of large numbers holds good for the sequence $\{X_k\}$ of independent random variables defined as

$$\Pr\{X_k = \pm 2^k\} = 2^{-(2k+1)} \Pr\{X_k = 0\} = 1 - 2^{-2k}$$

4. Answer any *three* of the following questions : 10×3=30

(a) If the variables are uniformly bounded, then the condition

$$\lim_{x \to \infty} \frac{Bx}{n^2} = 0$$

is necessary as well as sufficient for WLLN to hold. Prove this.

(b) Find the standard error of rth raw moment.

(c) State and prove Levy-Lindeberg central limit theorem.

(d) A candidate for election made a speech in city A but not in city B. A sample of 500 voters from city A showed that 59·6% of the voters were in favour of him, whereas a sample of 300 voters from city B showed that 50% of the voters favoured him. Discuss whether his speech could produce any effect on voters in city A. [Use 5% level]

(e) Define a Markov chain and an irreducible Markov chain. Classify the states of a Markov chain with examples.

(f) If $\{X_k\}$, $k = 1, 2, \ldots$ is a sequence of independent random variables each taking the values -1, 0, 1 and given that

$$P[X_k = 1] = P[X_k = -1], \quad P[X_k = 0] = 1 - \frac{2}{k}$$

examine if the laws of large number holds good for this sequence.

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