Total number of printed pages–7

1 (Sem–4/FYUGP) STA 41 MJ

2026

STATISTICS

(Major)

Paper : STA4400104MJ

(Probability–2 and Probability Distributions–2)

Full Marks : 60

Time : 2½ hours

The figures in the margin indicate full marks for the questions.

1. Answer the following questions as directed : 1×8=8

(a) If X is a random variable having probability function $f(x)$, then the function $\Sigma e^{itx} f(x)$, for 'i' be an imaginary unit, is known as

(i) Moment generating function

(ii) Probability distribution function

(iii) Characteristic function

(iv) None of the above

(Choose the correct option)

(b) If the mean of a gamma distribution is 3, what is its variance ?

(c) The sum of independent beta variates is also a beta variate.

(State True ***or*** *False)*

(d) The variance of a normal distribution is 9, find the value of quartile deviation.

(e) Characteristic function of a random variable always exists.

(State True ***or*** *False)*

(f) If X is a continuous random variable with mean 'μ' and variance 'σ^2', then for any positive number 'k'

$$P\{|X-\mu| \geq k\sigma\} < \frac{1}{k^2}$$

is known as

(i) Liapounoff's inequality

(ii) Markov's inequality

(iii) Chebyshev's inequality

(iv) None of the above

(Choose the correct option)

(g) Order statistics depend only on the order of the values, not the values themselves. *(State True* ***or*** *False)*

(h) The WLLN states that the sample mean converged in _____ to the theoretical mean as sample size 'n' increases indefinitely. *(Fill in the blank)*

2. Answer ***any six*** questions of the following : 2×6=12

(a) Define joint probability mass function and joint probability density function.

(b) Can $P(x) = \frac{2}{1+x}$ be the p.g.f. of a random variable X? Give reason.

(c) What is order statistics ? Mention *one* use of it.

(d) State Levy-Lindeberg theorem.

(e) Write the p.d.f. of Weibull distribution and state its mean.

(f) Under what condition negative binomial distribution may be regarded as the generalization of geometric distribution and how ?

(g) If X and Y are random variables then prove that

$$E[X] = E[E(X/Y)]$$

(h) Write the conditions for the existence of weak laws of large numbers (WLLN).

(i) If X follows exponential distribution with parameter 5. What are the value of the mean and variance of the distribution ?

(j) A random variable 'X' has mean 100 and variance 16. Using Chebyshev's inequality find the upper bound for $P(|X-100| \geq 8)$.

3. Answer ***any four*** of the following questions : 5×4=20

(a) Two ideal dice are thrown. If X_1 be the score on the first die, X_2 be the score on 2nd die and Y be the maximum of X_1 and X_2 the find

(i) Joint distribution of Y on X_1

(ii) $E[Y]$

(b) State and prove the additive property of Gamma distribution.

(c) If the random variable 'X' assumes the value 'r' with probability law

$P(X = r) = q^{r-1}p,\ r = 1, 2, 3, \ldots$

Find the m.g.f. of X and hence its mean and variance.

(d) Define Exponential distribution. Find its mean and variance.

(e) Derive the distribution of rth order statistics in taking a random sample of size 'n' from a continuous distribution.

(f) Define Hyper Geometric distribution and find its mean and variance.

(g) The p.d.f. of a random variable X is

$$f(x)=\frac{1}{\beta(m,n)}x^{m-1}(1-x)^{n-1};\ 0<x<1.$$

Find the distribution of $Y=\frac{1}{X}$.

(h) Define negative binomial distribution. Obtain its m.g.f. and c.g.f.

4. Answer ***any two*** questions from the following: 10×2=20

(a) (i) What is Geometric distribution? Explain its lack of memory property. 5

(ii) If X follows standard Cauchy distribution, show that X^2 follows

$$\beta_2\left(\frac{1}{2},\frac{1}{2}\right).$$ 5

(b) (i) State and prove Chebyshev's inequality. 7

(ii) If X is the number obtained in a throw of unbiased die, then using Chebyshev's inequality show that

$$P\{|X-\mu|\geq 2\cdot 5\}\leq 0\cdot 4667$$

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(c) State and prove the central limit theorem.

(d) The joint p.d.f. of the random variables X and Y is

$$f(x,y)=\begin{cases}\frac{2}{3}(x+2y) & ;\ \text{for } 0<x<1, 0<y<1\\ 0 & ;\ \text{otherwise}\end{cases}$$

7+3=10

Find the conditional density of X given $Y=y$ and use it to evaluate

$$P\left(x\leq\frac{1}{2}\middle/Y\leq 1/2\right)$$

(e) (i) Define characteristics function and state its important properties. Also state the uniqueness theorem of characteristic function. 5

(ii) Define Lognormal distribution. State its properties and uses. 5