# 2018

## STATISTICS

( Major )

Paper : 2.1

**( Numerical and Computational Techniques—I )**

*Full Marks* : 60

*Time* : 3 hours

*The figures in the margin indicate full marks for the questions*

**1.** Answer the following as directed : 1×7=7

*(a)* For positive integers $n$ and $m$

*(i)* $\Delta^n O^n = n!$ and $\Delta^m O^n = 0$, $n > m$

*(ii)* $\Delta^n O^n = 0$ and $\Delta^m O^n = 0$, $n < m$

*(iii)* $\Delta^n O^n = n!$ and $\Delta^m O^n = 0$, $n < m$

*(iv)* None of the above

( Choose the correct option )

( *Turn Over* )

(*b*) The value of $\Delta^{-1}(2)$ is

(*i*) 0

(*ii*) 2

(*iii*) $2x$

(*iv*) None of the above

( Choose the correct option )

(*c*) The relationship between the operators $\Delta$, $E$, $\delta$ is

(*i*) $\delta E \equiv \Delta$

(*ii*) $\delta E^{\frac{1}{2}} \equiv \Delta$

(*iii*) $\delta^{-1} E^{\frac{1}{2}} \equiv \Delta$

(*iv*) None of the above

( Choose the correct option )

(*d*) Consider the following statements :

*A* : Newton's formulae are applicable to nearly all cases of interpolation.

*B* : Newton's formulae do not converge as rapidly as central difference formulae.

(*i*) *A* is true but *B* is false

(*ii*) *A* is false but *B* is true

(*iii*) Both *A* and *B* are true

(*iv*) Neither *A* nor *B* is true

( Choose the correct option )

(e) Numerical differentiation is the process of evaluating the derivative(s) of a function at some particular value of the ____ when the values of the function corresponding to the given values of the independent variable are known.

( Fill in the blank )

(f) To derive Simpson's three-eighths rule from general quadrature formula, we assume that the integrand is a polynomial of ____ degree.

( Fill in the blank )

(g) The auxiliary equation of a third order difference equation has a repeated real root $\alpha$. Then the solution is

(i) $c_1\alpha^x + c_2\alpha^x$

(ii) $(c_1 + c_2 x)\alpha^x$

(iii) $(c_1 + c_2 x + c_3 x^2)\alpha^x$

(iv) $(c_1 + c_2 + c_3)\alpha^x$

( Choose the correct option )



( Turn Over )

2. Answer the following questions : 2×4=8

(a) Prove that

$$\Delta\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)\Delta f(x) - f(x)\Delta g(x)}{g(x)\; g(x+1)}$$

(b) Calculate the value of $\Delta^2 O^5$.

(c) Solve the difference equation

$$x_n = x_{n-1} + x_{n-2}$$

(d) Show that

$$\int_0^1 u_x dx = \frac{1}{12}(5u_1 + 8u_0 - u_{-1})$$

3. Answer any *three* of the following questions : 5×3=15

(a) If $n$ is a positive integer, then show that

$$E^n y_x = \sum_{k=0}^{n} \binom{n}{k} \Delta^k y_x$$

and hence

$$E^3 y_x = y_x + 3\Delta y_x + 3\Delta^2 y_x + \Delta^3 y_x$$

(b) Prove that

$$\Delta^n 0^{n+1} = \frac{1}{2} n(n+1)\,!$$

(c) State and prove Gauss's forward formula for equal intervals.

(d) Solve the difference equation

$$u_{x+2} - 4u_{x+1} + 4u_x = 2^x$$

(e) If α, β be the roots of $x^2 + ax + b = 0$, show that the iteration

$$x_{n+1} = -\frac{(ax_n + b)}{x_n}$$

will converge near $x = \alpha$, if $|\alpha| > |\beta|$ and the iteration

$$x_{n+1} = \frac{-b}{(x_n + a)}$$

will converge near $x = \alpha$, if $|\alpha| < |\beta|$.

4. Answer the following questions : 10×3=30

(a) Show that for the interpolation of $f(x)$ relative to 0, α, 1, Lagrange's formula gives approximately

$$f(x)=\left[1-\frac{x(x-\alpha)}{1-\alpha}\right]f(0)+\frac{x(1-x)}{1-\alpha}\,\frac{f(\alpha)-f(0)}{\alpha}+\frac{x(x-\alpha)}{1-\alpha}f(1)$$

Also show that if $\alpha \to 0$, it reduces to

$$f(x)=(1-x^2)f(0)+x(1-x)f'(0)+x^2 f(1)$$

*Or*

Solve the following difference equations :

(i) $u_{x+3}-5u_{x+2}+8u_{x+1}-4u_x=x\cdot 2^x$

(ii) $u_{x+2}+a^2u_x=\cos ax$

(b) If $f(x)$ is a function whose fifth differences are constants and if $\int_{-1}^{1} f(x)\,dx$ can be expressed in the form of $pf(-\alpha)+q(0)+p(\alpha)$, find $p$, $q$ and α. Use the formula to obtain the value of $\log_e 2$ to four decimal places from the integral $\int_0^1 \frac{dx}{1+x}$. [Given $\sqrt{0\cdot15}=0\cdot3873$]

*Or*

Derive Euler-Maclaurin summation formula.

(c) Using Newton's formula, derive a recurrence relation to find the cube root of $N$. Using this relation, evaluate $(10)^{\frac{1}{3}}$.

*Or*

Write a note on inverse interpolation. Apply Lagrange's formula (inversely) to find a root of the equation $f(x) = 0$, when $f(30) = -30$, $f(34) = -13$, $f(38) = 3$, $f(42) = 18$.

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