2018

STATISTICS

(Major)

Paper : 2.2

(Mathematical Method—I)

Full Marks : 60

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. Answer the following questions : 1×7=7

(a) What is the value of $\Gamma\left(\frac{3}{2}\right)$?

(b) Is it true that the range of a sequence always infinite?

(c) Write that type of beta integral where the integrand is a ratio of two algebraic functions.

(d) Write 'true' or 'false' on the following statement :

"The minimum value of the function $f(x) = e^{-x}$, $0 < x < \infty$ is 1."

(e) Give an example of a sequence which is oscillatory.

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(f) Define $\frac{\partial}{\partial x} f(x, y)$ and $\frac{\partial}{\partial y} f(x, y)$.

(g) State a necessary and sufficient condition for a sequence to be convergent.

2. Answer the following questions : 2×4=8

(a) Give the geometrical interpretation of Rolle's theorem.

(b) If $x = r\cos\theta$ and $y = r\sin\theta$, find the Jacobian of the transformations of variables (x, y) to (r, θ).

(c) Clearly state the mean value theorem for three functions $f(x)$, $g(x)$ and $h(x)$ of a variable x.

(d) Show that the sequence $\{\cos n\pi\}$, $n \in N$ is bounded. Is it convergent?

3. Answer any *three* questions : 5×3=15

(a) If $x = r \cdot \sin\theta \cdot \cos\phi$, $y = r \cdot \sin\theta \cdot \sin\phi$ and $z = r \cdot \cos\theta$, then show that

$$\frac{\partial(x, y, z)}{\partial(r, \theta, \phi)} = r^2 \sin\theta$$

(b) Examine the convergence either of the following :

(i) $\int_0^\infty \frac{x}{(1+x)^3} dx$

(ii) $\int_0^\infty \frac{x^2}{(1+x)^3} dx$

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(c) Find the maxima and minima, if exist, of the function $y = 3\cos^2\theta + \sin^6\theta$, $-\frac{\pi}{2} < x < \frac{\pi}{2}$.

(d) State Taylor's series for single variable and find the series expansion of $\sin x$.

(e) State and prove a necessary and sufficient condition for uniform convergence of a sequence.

4. Answer any *three* questions : 10×3=30

(a) State and prove Cauchy's mean value theorem and derive Lagrange's mean value theorem from it. 2+5+3=10

(b) Prove the following : 2+2+6=10

(i) $\Gamma(n+1) = n!$ if n is positive integer

(ii) $\beta(m, n) = \beta(n, m)$

(iii) $\beta(m, n) = \dfrac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$

(c) Define Jacobian. If $u = x^2 + y^2 + z^2$, $v = x + y + z$ and $w = xy + yz + zx$, then show that

$$\frac{\partial(u, v, w)}{\partial(x, y, z)}$$

vanishes identically and $v^2 = u + 2w$. 2+5+3=10

(Turn Over)

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(d) Define limit and continuity of a function of two variables. Examine the continuity of the function

$$f(x, y) = \begin{cases} \dfrac{xy}{\sqrt{x^2 + y^2}} & , (x, y) \neq (0, 0) \\ 0 & , (x, y) = (0, 0) \end{cases}$$

at $(x, y) = (0, 0)$. 2+2+6=10

(e) Given

$$f(x, y) = \begin{cases} \dfrac{x^2 y(x^3 - y^3)}{(x^3 + y^3)} & , (x, y) \neq (0, 0) \\ 0 & , (x, y) = (0, 0) \end{cases}$$

Compute $f_x(0, 0)$, $f_{xx}(0, 0)$, $f_{xy}(0, 0)$, $f_{yx}(0, 0)$ and $f_{yy}(0, 0)$, if exist. 10

(f) Examine the uniform convergence of—

(i) the sequence $\left\{\dfrac{n^2 x}{1 + n^3 x}\right\}$ in $[0, 1]$

(ii) the series $\displaystyle\sum_{n=1}^{\infty} \frac{\sin nx}{n^p}$, $p > 1$ 10

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