2019

COMPUTER SCIENCE

(Major)

Paper : 6.1

(Automata Theory and Languages)

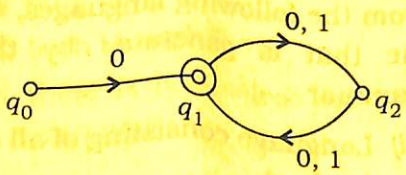
Full Marks : 60

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. For the following questions, choose the correct answer from the choices given below them : 1×7=7

(a) Consider the following transition diagram of a finite automaton :



Here q_0 is the starting state.

From the following strings, select the string that is accepted by it :

(i) 1011 *(ii)* 01011

(iii) 101 *(iv)* 0011

(b) Regular expression for the language consisting of all strings in $\{a, b\}$ ending with b is

(i) a^*b^* (ii) $(a+b)^*b$

(iii) a^*b (iv) $a^*b^*a^*b$

(c) From the following context-free grammars, select the one that is in Chomsky Normal Form (CNF) :

(i) $S \to ABa$
$A \to Aa$
$B \to Bb$

(ii) $S \to aS$
$S \to bS$
$S \to \varepsilon$

(iii) $S \to aSb$
$S \to c$

(iv) $S \to YS \mid x$
$Y \to SY \mid y$

(d) Consider the following grammar :

$S \to aSb$
$S \to c$

From the following languages, select the one that is generated by the above grammar :

(i) Language consisting of all strings in $\{a, b, c\}$

(ii) $L = \{a^n cb^n \mid n > 0\}$

(iii) $L = \{a^n cb^n \mid n \geq 0\}$

(iv) Language represented by the regular expression a^*cb^*

(e) The nullable variables in the following grammar

$S \to a \mid Xb \mid aYa$
$X \to Y \mid \varepsilon$
$Y \to b \mid X$

are

(i) X only
(ii) both X and Y
(iii) Y only
(iv) It has no nullable variables

(f) Useless variables in the following grammar

$S \to A$
$A \to aA \mid \varepsilon$
$B \to bA$

are

(i) only B
(ii) only A
(iii) both A and B
(iv) there is no useless variable in the grammar

(g) If r_1 and r_2 are two regular expressions, then which one of the following is not a regular expression?

(i) $r_1 - r_2$ (ii) $r_1 + r_2$

(iii) $r_1 r_2$ (iv) r_1^*

2. Answer the following questions : 2×4=8

(a) Consider the following regular expressions :

(i) $(a+b)^* \; a(a+b)^* \; a(a+b)^*$

(ii) $b^* ab^* a(a+b)^*$

(iii) $b^* a(a+b)^* ab^*$

(iv) $b^* ab^* ab^*$

Select the regular expression that does not represent the language consisting of all strings in a and b having at least two a's. What language does this particular regular expression represent?

(b) Draw the transition diagram of a finite automaton that accepts the language consisting of all strings in a and b ending with the substring 'aab'.

(c) Convert the following grammar to Chomsky Normal Form (CNF) :

$$S \rightarrow AS \mid AAS$$
$$A \rightarrow SA \mid aa$$

(d) State the pumping lemma for regular languages and cite one use of it.

3. Answer any *three* of the following questions : 5×3=15

(a) Prove that the set of context-free languages is closed under union.

(b) Prove that the language $L = \{a^n b^n c^n \mid n \geq 0\}$ is not a context-free language.

(c) Design a finite automaton for the language consisting of all strings in $(a+b)^*$ not having 'aab' as a substring.

(d) When is a CFG said to be ambiguous? Show that the following grammar

$$S \rightarrow SS$$
$$S \rightarrow \varepsilon$$
$$S \rightarrow aSb \mid bSA$$

is ambiguous.

(e) Remove all useless symbols, null productions and unit productions from the following grammar :

$$S \rightarrow aA \mid aBB$$
$$B \rightarrow aaA \mid \varepsilon$$
$$B \rightarrow bB \mid bbC$$
$$C \rightarrow B$$

2.

4. Answer any *three* of the following questions : 10×3=30

(a) Prove that if a language L is accepted by a DFA, then it is described by a regular expression.

(b) *(i)* Give a context-free grammar that generates the language

$$L = \{w \in (a+b)^* \ / \ w = w^R$$

i.e., w is a palindrome}

(ii) Consider the following CFG, G :

$$S \to aSA \mid aAA \mid b$$
$$A \to bBBB$$
$$B \to b$$

Construct a PDA accepting $L(G)$.

(c) Construct a pushdown automaton (PDA) for any *one* of the following languages :

(i) $L = \{a^n b^{2n} \ / \ n \geq 0\}$

(ii) $L = \{ww^R \ / \ w \in (a+b)^*\}$

(d) If L is a CFL, then prove that there is a PDA that accepts L.

(e) Prove the equivalence of DFA and NFA.

(f) What is a parse tree? What are leftmost and rightmost derivations in a grammar? Give examples. Consider the following grammar :

$$E \to T$$
$$T \to F$$
$$F \to I$$
$$E \to E + T$$
$$T \to T * F$$
$$F \to (E)$$
$$I \to a \mid b \mid c$$

where E is the starting variable. Construct a derivation tree for

$$((a+b) * c) + a$$

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