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STATISTICS

(Major)

Paper : 3·2

(Distribution—I)

Full Marks : 60

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. Answer the following questions : 1×7=7

(a) Under what condition, binomial distribution reduces to Bernoulli distribution?

(b) Does the difference of two independent Poisson variates follow Poisson distribution?

(c) Under what condition, negative binomial distribution may be regarded as the generalization of geometric distribution?

(d) Under what conditions, hypergeometric distribution tends to binomial distribution?

(Turn Over)

(e) If X_i, $(i = 1, 2, \cdots, n)$ are i.i.d. standard Cauchy variate, write the distribution of

$$\bar{X} = \frac{1}{n}\sum_{i=1}^{n} X_i$$

(f) Fill in the blank :

If $X \sim N(\mu, \sigma^2)$, then

$$P(\mu - 3\sigma < X < \mu + 3\sigma) = \_\_\_\_.$$

(g) Define exponential distribution.

2. Answer the following questions : 2×4=8

(a) Show that the m.g.f. of binomial distribution tends to the m.g.f. Poisson distribution as $n \to \infty$.

(b) Show that for the negative binomial distribution

$$P(x) = \binom{-r}{x} Q^{-r}\left(-\frac{P}{Q}\right)^x ; \; x = 0, 1, 2, \cdots, \; Q - P = 1$$

cumulant generating function is

$$-r\log[1 - P(e^t - 1)]$$

(c) If X and Y are two independent unit normal variates, find the probability density function of $\frac{X-Y}{\sqrt{2}}$.

(Continued)

(d) A random sample of size n, $x_1, x_2, \cdots, x_n$ is drawn from the population

$$dP(x) \frac{1}{\Gamma(n)} e^{-x} x^{n-1} ; \; 0 < x < \infty$$

If $\bar{x}$ is mean of the sample, find the distribution of $\bar{x}$.

3. Answer any *three* of the following : 5×3=15

(a) If X follows binomial distribution with parameters n and p, and if μ_r and μ_{r+1} exist, then prove that

$$\mu_{r+1} = pq\left[nr\mu_{r-1} + \frac{d\mu_r}{dp}\right]$$

Hence find μ_2 and μ_3.

(b) Show that hypergeometric distribution tends to binomial distribution under certain conditions.

(c) Discuss the importance of normal distribution in statistics.

(d) Prove that the quotient of two independent gamma variates with parameter l and m is $\beta(l, m)$ variate of 2nd kind.

(Turn Over)

(e) If $X_1, X_2, \cdots, X_n$ are independent random variables having an exponential distribution with parameters $\theta_1, \theta_2, \cdots, \theta_n$ respectively, then prove that

$$Z = \min(X_1, X_2, \cdots, X_n)$$

has exponential distribution.

4. Answer any *three* of the following : 10×3=30

(a) *(i)* If X is a Poisson variate with parameter m and Y is another discrete variable whose conditional distribution for a given X is given by

$$P(Y = r \mid X = x) = \binom{x}{r} P^r (1-P)^{x-r}$$

$0 < P < 1, \quad r = 0, 1, 2, \cdots, x$; then show that the unconditional distribution of Y is a Poisson distribution with parameter mp. 6

(ii) If the random variable X follows uniform distribution on $[-\frac{\pi}{2}, \frac{\pi}{2}]$, find the p.d.f. of $Y = \tan X$. 4

(b) *(i)* Derive the probability mass function of the negative binomial distribution. Also obtain m.g.f. of negative binomial distribution and hence show that mean is less than variance. 7

(ii) State the application of hypergeometric distribution. 3

(c) State and prove the relationship between binomial distribution and normal distribution. 10

(d) *(i)* Let X be a discrete random variable having geometric distribution with parameter p. Obtain its mean and variance. Also show that for any two positive integers s and t

$$P(X > s + t \mid X > s) = P(X > t)$$ 7

(ii) Write the applications of beta distribution. 3

(e) *(i)* Find the characteristic function of standard Laplace distribution and hence find its mean and standard deviation. 4

(ii) Find the moment generating function of the normal distribution $N(\mu, \sigma^2)$ and deduce that

$$\mu_{2n+1} = 0$$

$$\text{and } \mu_{2n} = 1 \cdot 3 \cdot 5 \cdot \cdots (2n-1)\sigma^{2n}$$

where μ_n denotes the nth central moment. 6

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