2019

STATISTICS

(Major)

Paper : 6.4

(Computer Programming and Multivariate Analysis)

Full Marks : 60

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. Answer the following as directed : 1×7=7

(a) Let $X \sim N_3 (\mu, \Sigma)$ with

$$\Sigma = \begin{pmatrix} 4 & 1 & 0 \\ 1 & 3 & 0 \\ 0 & 0 & 2 \end{pmatrix}$$

Are X_1 and X_3 independent?

(b) State the built-in mathematical function in Fortran 77 to find the square root of x.

(c) Marginal distribution of any x_j of a multinomial distribution with parameters $(n, p_1, p_2, ..., p_k)$ follows binomial distribution. (State True or False)

(*Turn Over*)

(d) Let $(X, Y) \sim$ BVND $(\mu_x, \mu_y, \sigma_x^2, \sigma_y^2, \rho)$. Then the conditional variance of $X/Y = y$ is ____. (Fill in the blank)

(e) Write the decimal equivalent of octal number 2534_8.

(f) What is the value of M in the following Fortran 77 expression?

$$M = 2 * 7 / 5$$

   (i) $M = 2 \cdot 8$

   (ii) $M = 3$

   (iii) $M = 2$

   (iv) None of the above

         (Choose the correct option)

(g) Define Hotelling's T^2 statistic.

2. Answer the following questions : $2 \times 4 = 8$

(a) State any two properties of multivariate normal distribution.

(b) Write equivalent FORTRAN 77 statements for each of the following expressions :

   (i) $e^{-x} x^{kx}$

   (ii) $e^{-\left(\frac{x-a}{b}\right)^2}$

(c) Let $X \sim N_p (\mu, \Sigma)$. Then find var (CX) where C is a $p \times p$ non-singular matrix.

(d) Write an algorithm to find the arithmetic mean of three numbers A, B, C.

                           (Continued)

3. Answer any *three* of the following questions : $5 \times 3 = 15$

(a) Let $(X, Y) \sim$ BVND $(0, 0, 1, 1, \rho)$. Then show that

$$Q = \frac{X^2 - 2\rho XY + Y^2}{(1 - \rho^2)}$$

is distributed as chi-square with $n = 2$ d.f.

(b) What is the final value of a in the following sequence of statements in FORTRAN 77?

$$a = 2 \cdot 45$$
$$a = (a + 0 \cdot 06) * 10$$
$$k = a$$
$$a = k$$
$$a = a / 10 \cdot 0$$

If $a = 2 \cdot 45$ is replaced by $a = 2 \cdot 43$ above, what is the final value of a ? $3 + 2 = 5$

(c) Obtain the probability-generating function of multinomial distribution with parameters $(n, p_1, p_2, \cdots p_k)$.

(d) Draw a flowchart to find the largest among three numbers M, N, P.

(e) Examine if Hotelling's T^2 is invariant under changes in the unit of measurement.

                           (Turn Over)

4. Answer the following questions :   10×3=30

(a) State the pdf of BVND $(\mu_1, \mu_2, \sigma_1^2, \sigma_2^2, \rho)$.
Derive bivariate normal density as a particular case of multivariate normal distribution.   1+9=10

Or

(b) If $X \sim N_p(\mu, \Sigma)$, then prove that the quadratic form in the multivariate normal density function

$$Q = (X - \mu)' \Sigma^{-1} (X - \mu)$$

follows χ^2 distribution with pdf.

(c) Write a FORTRAN 77 program to find the regression coefficient of Y on X.

Or

(d) (i) Write an explanatory note on 'Arithmetic IF' statement used in FORTRAN 77.

(ii) Explain briefly about WHILE–DO statement.   5+5=10

(e) Derive mean and variance of multinomial distribution. Also compute the variance, covariance matrix Σ.   4+4+2=10

Or

(f) (i) Let $X \sim N_5(\mu, \Sigma)$. Then find the distribution of $(X_2 \ X_4)'$.

(ii) Write a FORTRAN 77 program to calculate harmonic mean of n observations $x_1, x_2, \cdots, x_n$.   4+6=10

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