2018

STATISTICS

(Major)

Paper : 3.1

(Mathematical Methods—II)

Full Marks : 60

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. Answer the following as directed : 1×7=7

   (a) Write down the quadratic form corresponding to the matrix

   $$D = \text{diag}(\lambda_1, \lambda_2, \cdots, \lambda_n)$$

   (b) Consider the sample observations $x_1, x_2, \cdots, x_n$ drawn from a population. Express sample mean as the product of two matrices.

(c) The var-covariance matrix of the multivariate normal distribution is

(i) positive definite

(ii) positive semi-definite

(iii) negative definite

(iv) negative semi-definite

(Choose the correct option)

(d) Consider the system of homogeneous linear equations

$$(A)_{m\times n}(X)_{n\times 1}=(0)_{m\times 1}$$

and suppose $P(A)=r$. Find out the number of linearly independent solutions for this system of equations.

(e) Consider the following system of non-homogeneous linear equations :

$$(A)_{m\times n}(X)_{n\times 1}=(B)_{m\times 1}$$

State the condition when this system of equations will permit a unique solution.

(f) Suppose that A is a (5×5) matrix and $P(A)=4$. Find $\det(A)$.

(g) A is a non-singular matrix and $\det(A)=35$. Find $\det(A^{-1})$.

2. Answer any *four* of the following questions : 2×4=8

(a) Consider the matrix

$$A=\begin{pmatrix}0 & 2 & 3\\ 2 & 4 & 0\\ 3 & 0 & 1\end{pmatrix}$$

Obtain the appropriate elementary matrix E that transforms A to

$$\tilde{A}=\begin{pmatrix}0 & 2 & 5\\ 2 & 4 & 6\\ 3 & 0 & 4\end{pmatrix}$$

(b) Consider $X=\begin{pmatrix}x_1\\ x_2\\ x_3\end{pmatrix}$ to be a sample of 3 observations drawn from a population. Consider the matrix

$$M=\begin{pmatrix}2/3 & -1/3 & -1/3\\ -1/3 & 2/3 & -1/3\\ -1/3 & -1/3 & 2/3\end{pmatrix}$$

Show that sample variance

$$s^2=\frac{1}{3}\sum_{i=1}^{3}(x_i-\bar{x})^2$$

can be expressed as $Q=\frac{1}{4}X'\cdot MX$.

(c) Given A and B are square matrices of the same order and let $\text{adj}\,B = C$ and $\text{adj}\,A = D$ (C and D are matrices of appropriate order). Obtain $\text{adj}(AB)$ in terms of matrices C and D.

(d) Given for a (3×3) matrix A, $\det(\text{adj } A) = 20$. Find $\det(A)$.

(e) Differentiate between an orthogonal matrix and an unitary matrix.

3. Answer any *three* of the following questions : 5×3=15

(a) Show that the matrix

$$\sigma_y = \begin{pmatrix} 0 & -p \\ p & 0 \end{pmatrix}$$

is unitary.

(b) Suppose A is a non-zero column matrix as

$$A = \begin{pmatrix} a_{11} \\ a_{21} \\ a_{31} \end{pmatrix} \text{ and } B = (b_{11}, b_{12}, b_{13})$$

is a non-zero row matrix. Obtain AB and argue that $P(AB) = 1$.

(c) Find the rank of the following matrix :

$$A = \begin{pmatrix} 2 & 3 & 7 \\ 3 & -2 & 4 \\ 1 & -3 & -1 \end{pmatrix}$$

(d) Compute the inverse of the following matrix by using elementary operations :

$$\begin{pmatrix} 2 & -1 & 3 \\ 1 & 1 & 1 \\ 1 & -1 & 1 \end{pmatrix}$$

(e) Prove that a necessary and sufficient condition that values; not all zero, may be assigned to the n variables $x_1, x_2, \cdots, x_n$ so that the n homogeneous equations

$$a_{i1}x_1 + a_{i2}x_2 + \cdots + a_{in}x_n = 0 \; (i = 1, 2, \cdots, n)$$

hold simultaneously, is that the determinant of $A = (a_{ij})_{n \times n}$ is equal to zero.

4. Answer any *three* of the following questions : 10×3=30

(a) Consider the following quadratic form : 10

$$X'AX = 6X_1^2 + 3X_2^2 + 3X_3^2 - 4X_1X_2 - 2X_2X_3 + X_1X_3$$

Reduce it to the canonical form

$$a_1Y_1^2 + a_2Y_2^2 + a_3Y_3^2$$

by a set of non-singular transformations $X = PY$, where

$$X = \begin{pmatrix} X_1 \\ X_2 \\ X_3 \end{pmatrix}, \; Y = \begin{pmatrix} Y_1 \\ Y_2 \\ Y_3 \end{pmatrix}$$

and P is a (3×3) non-singular matrix.

(b) Find the adjoint of the matrix

$$A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{pmatrix}$$

Hence verify

$$A(\text{adj}\, A) = (\text{adj}\ A)A = |A| I_n$$ 10

(c) Consider the following system of non-homogeneous linear equations :

$$(A)_{m \times n} (X)_{n \times 1} = (B)_{m \times 1}$$

Show that this system of equations is consistent if and only if the coefficient matrix A and the augmented matrix $(A : B)$ are of the same rank. 10

(d) Suppose

$$X = \begin{bmatrix} 1 & 2 \\ 0 & -1 \\ 3 & 1 \end{bmatrix}$$

Evaluate $M = I - X(X'X)^{-1} X'$, where the notations have their usual meanings. Show that $M = M^2$ and find the rank of M and M^2. 10

(e) (i) Given the matrix

$$M = \begin{pmatrix} 5 & 4 & 1 \\ 3 & 5 & 2 \\ 4 & 9 & 7 \\ 6 & 2 & 8 \end{pmatrix}$$

Obtain all possible minors of M.

(ii) Consider the three linearly independent vectors a_1, a_2 and a_3. Suppose

$$b_1 = a_1 + a_2$$
$$b_2 = a_2 + a_3$$

and then consider the matrix

$$A = (a_1, a_2, a_3, b_1, b_2)$$

Find $P(A)$. 6+4=10

(f) Show that the following system of equations are consistent and hence solve them : 10

$$3x+7y+5z=4$$
$$26x+2y+3z=9$$
$$2x+10y+7z=5$$

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